

THE MOTION AND DISTRIBUTION OF THE SUN-SPOTS

A STATISTICAL INVESTIGATION

BY

O. A. ÅKESSON

WITH 8 FIGURES AND 1 PLATE



LUND

C. W. K. GLEERUP

LEIPZIG

OTTO HARRASSOWITZ

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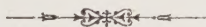


BY

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LIC. PHIL. KR.

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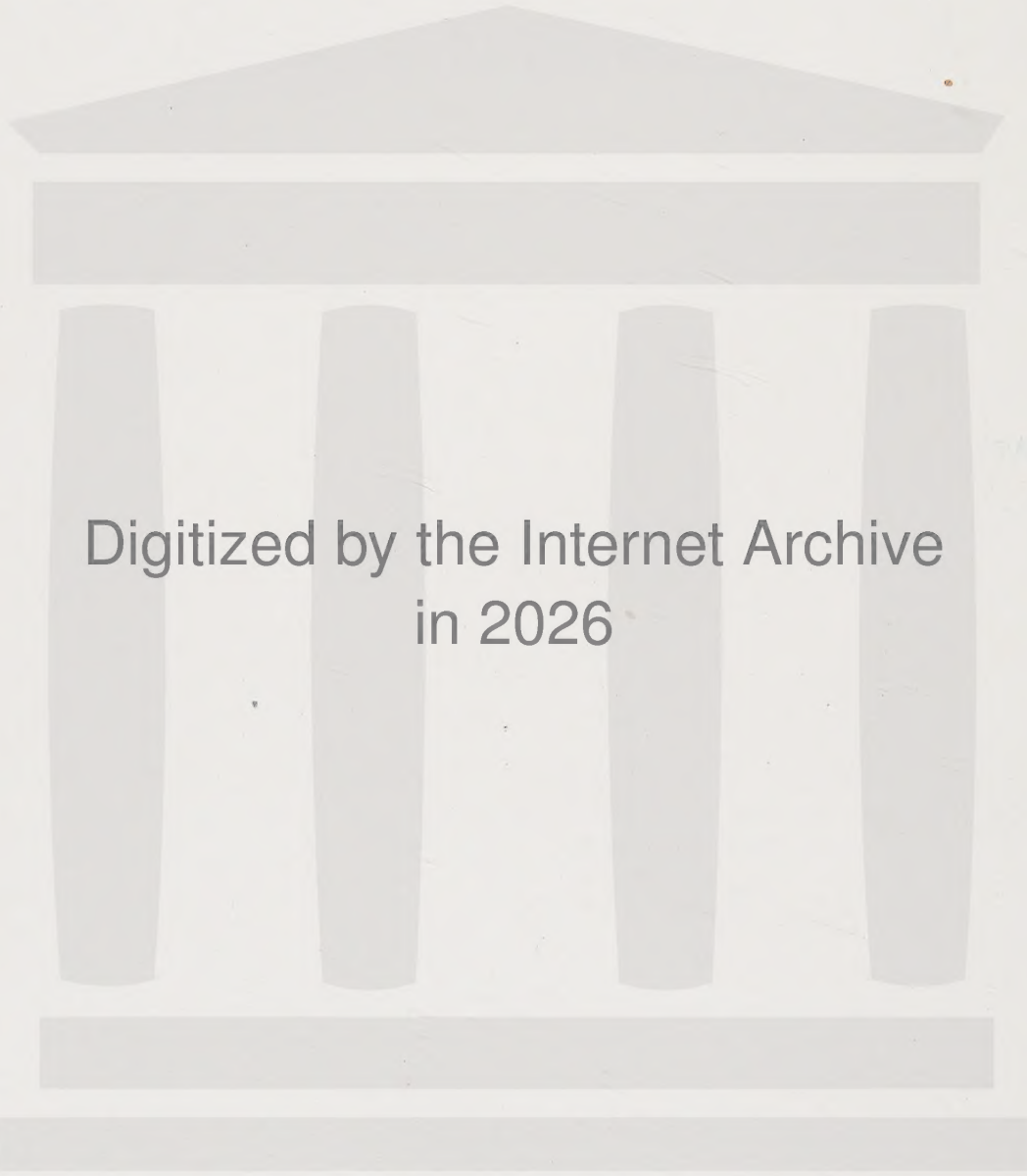


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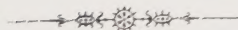
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Introduction.

Historical Summary.

In this section I shall give a short account of the researches into the *periodicity*, *motion* and *distribution* of the sun-spots. Consequently spectral-analytical investigations will not be discussed, however important and decisive these researches have been for our ideas of the solar phenomena. In connection with the investigations of WOLF I give the result of some numerical computations, which I have made in order to determine the relation between the relative numbers and the sun-spot area.

A study of the motion etc. of the sun-spots was not possible until the discovery of the telescope in the year 1608, though sun-spots had been observed at different times long before¹. From the observations of GALILEI, SCHEINER and FABRICIUS it soon became evident that the spots were formations on the sun, and that their motion could be easily explained by the rotation of the sun about an axis, inclining to that of the ecliptic. SCHEINER found this inclination to be $7\frac{1}{2}^{\circ}$, the longitude of the ascending node of the sun's equator on the ecliptic $69\frac{1}{2}^{\circ}$, and the rotation period 25 days. These values deviate very little from values found later.

The Periodicity of the Sun-spots. On the whole, no systematic observations of the sun-spots were made during the seventeenth and eighteenth centuries. The observations that were made, however, showed that the number of sun-spots was subject to great variations. It was the Danish astronomer CHRISTIAN HORREBOW who first noticed that these variations are periodic. Of his sun-spot counts, only those for the years 1761 and 1764—1777 remain, although they were started as early as 1738, part of the material he had collected being destroyed during the bombardment of Copenhagen in 1807¹. These observations of HORREBOW being of some interest, I borrow Table I from the paper of THIELE. In the diary of HORREBOW from 1775—1776 we find the following remark: »Obwohl sich aus den Beob-

¹ J. WILLIAMS: Chinese observations of solar spots from A. D. 301 to 1205, extracted from the encyclopædia of Ma Twan Lin. Monthly Notices XXXIII.

² TH. N. THIELE: De macularum Solio antiquioribus quibusdam observationibus Hafniæ institutis. Astronomische Nachrichten No 1193.

achtungen ergibt, dass die Veränderungen und Wechsel der Sonnenflecken häufig sind, so kann doch keine bestimmte Regel dafür gefunden werden, nach welcher Ordnung und nach wieviel Jahren dieser Wechsel vollzieht. Dieses kommt hauptsächlich davon her, dass die Astronomen sich bisher wenig bemühten, häufige Sonnenfleckenbeobachtungen zu machen, ohne Zweifel weil sie glaubten, es gehe daraus nichts hervor, das für die Astronomie oder Physik grosses Interesse hätte. Es ist indessen zu hoffen, dass man durch eifriges Beobachten auch hier eine *Periode* auffinden werde wie in den Bewegungen der übrigen Himmelskörper; dann erst wird es Zeit sein, zu untersuchen in welcher Weise die Körper, die von der Sonne getrieben und beleuchtet sind, durch die Sonnenflecken beeinflusst werden»¹.

TABLE I.
Horrebow's Sun-spot Observations.

Year	Daily Spot-number	Days without Spots	Days of Observations
1761	6.40	2	75
1762	—	—	—
1763	—	—	—
1764	2.20	6	40
1765	0.20	4	5
1766	0.10	59	62
1767	3.14	16	132
1768	5.42	4	159
1769	10.85	0	123
1770	9.12	0	131
1771	7.46	4	120
1772	5.26	0	78
1773	2.19	27	124
1774	1.80	39	101
1775	0.46	123	167
1776	1.23	81	178
1777	6.05	1	65

TABLE II.
Schwabe's Observations 1826—43.

Year	Number of Spot-groups	Days without Spots	Days of Observations
1826	118	22	277
1827	161	2	273
1828	225	0	283
1829	199	0	244
1830	190	1	217
1831	149	3	239
1832	84	49	270
1833	33	139	267
1834	51	120	273
1835	173	18	244
1836	272	0	200
1837	333	0	168
1838	282	0	202
1839	162	0	205
1840	152	3	263
1841	102	15	283
1842	68	64	307
1843	34	149	324

HORREBOW's observations were, however, forgotten until 1859, when THIELE published the summary mentioned above, but by that time the problem of the periodicity was solved.

In the year 1826, HEINRICH SCHWABE, German apothecary of *Dessau*, began to make systematic observations on the sun-spots, which he continued up to 1868. In the »*Astronomische Nachrichten*» for 1844, SCHWABE gives a summary of the observations made up to that time. From these observations there appears an evident variation in the number of sun-spots, during a period that SCHWABE supposed to be 10 years. In Table II the result of SCHWABE's observations are given. Of interest is a communication from SCHWABE to CARRINGTON 1857, where he mentions that it was HARDING of *Göttingen* who induced him to begin these solar observations, which, according to HARDING, »might be rewarded by the discovery of

¹ From WOLF: *Handbuch der Astronomie* II. 408.

a planet interior to Mercury». As is well known, SCHWABE did not succeed in discovering a planet interior to Mercury. That he valued the discovery of the periodicity of the sun-spots is shown in the above mentioned letter, where he compares himself with Saul, »who went out to seek his father's asses, and found a kingdom».

Induced by the discovery of SCHWABE, WOLF of Zürich made his well-known researches. Up to that time, they had employed the number of spots, or groups of spots, as a measure of the frequency of sun-spots. For well-known reasons, WOLF, in the year 1851, introduces his *relative numbers* r , defined by the relation

$$r = k(10g + f)$$

where

g = the number of spot groups and individual spots,

f = the sum of individual spots and spots belonging to the different groups,

k = a constant, depending on the power of the instrument.

For the observations made by WOLF with his refractor of 3 inches, he put $k = 1$.

Though these relative numbers are arbitrarily chosen, they are, nevertheless, a much better measure of the spot-activity than the preceding spot-numbers. In general one assumes the relative numbers to be proportional to the spot area. »One hundred as a sun-spot number corresponds to about $\frac{1}{500}$ of the sun's visible disk covered by spots, including both umbras and penumbras.»¹

In order to examine the relation between relative numbers and spot-areas, I have used the values of the daily mean area for every year, corrected for shortening, which values are obtained from photographs of the sun published in the Greenwich Observations². This examination comprises the time from 1874 to 1910. The daily mean area, expressed in millionths of the sun's visible disk, I designate by Δ . The relative numbers of WOLF are, as above, designated by r . In Table III, r and Δ are given from 1874—1910.

TABLE III.
The Relative Numbers and
Spot-areas 1874—1910.

Year	r	Δ
1874	44.7	601
1875	17.1	272
1876	11.3	122
1877	12.3	92
1878	3.4	24
1879	6.0	49
1880	32.3	416
1881	54.3	730
1882	59.7	1002
1883	63.7	1155
1884	63.5	1079
1885	52.2	811
1886	25.4	381
1887	13.1	179
1888	6.8	89
1889	6.3	78
1890	7.1	99
1891	35.6	569
1892	73.0	1214
1893	84.9	1464
1894	78.0	1282
1895	64.0	974
1896	41.8	543
1897	26.2	514
1898	26.7	375
1899	12.1	111
1900	9.5	75
1901	2.7	29
1902	5.0	62
1903	24.4	340
1904	42.0	488
1905	63.5	1191
1906	53.8	778
1907	62.0	1082
1908	48.5	697
1909	43.9	692
1910	18.6	264

¹ ABBOT: The Sun p. 184.

² A more complete account of these photographs will be given in Chapter I.

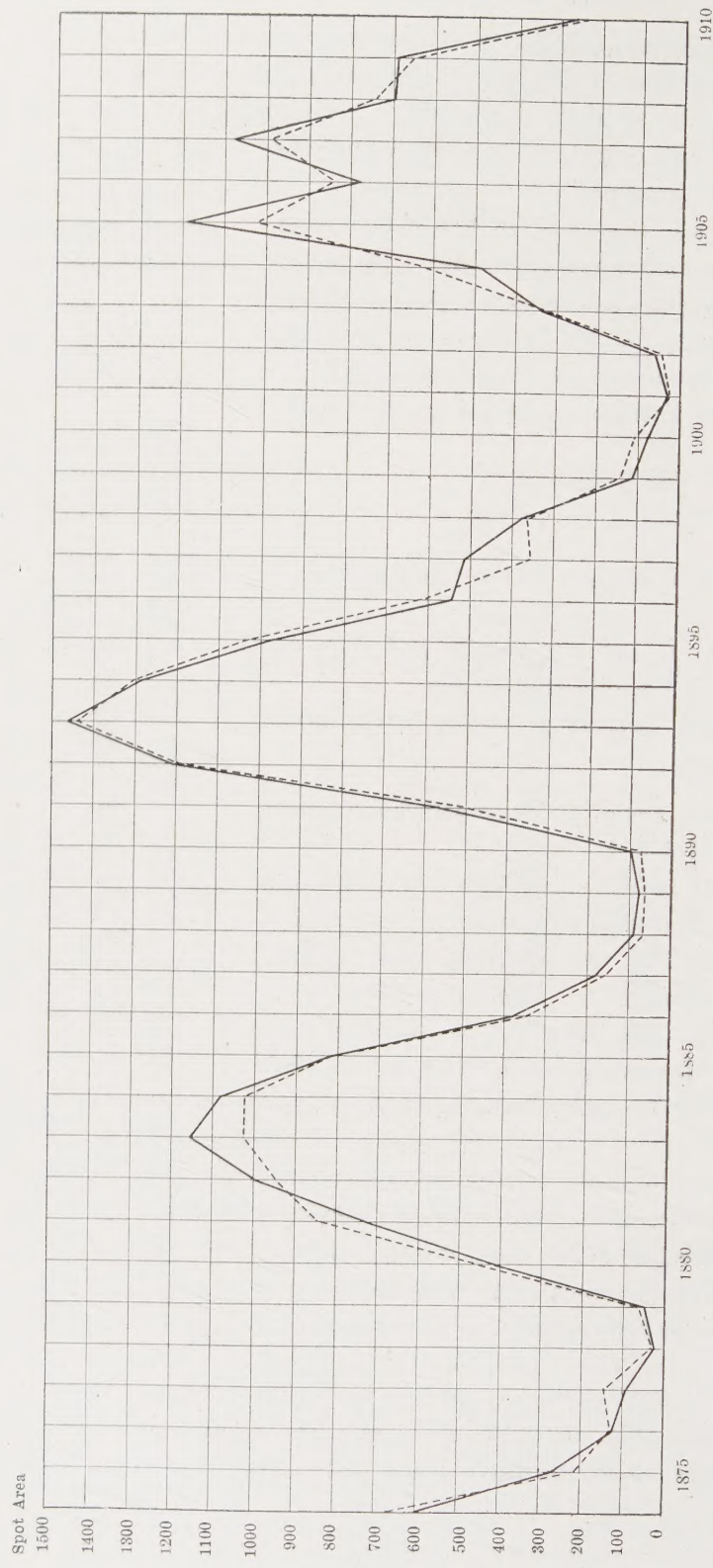


Fig. 1. Daily Mean Area of Sun-spots 1874—1910 expressed in Millionths of the Sun's visible Disk.

— From Solar Photographs
 - - - Computed from the Relative Numbers.

The simplest assumption is

$$\Delta = k_0 \cdot r$$

where k_0 is a constant that may be determined from 37 equations of condition. Next I supposed

$$\Delta = k_1 + k_2 \cdot r$$

which assumption gave a somewhat better result than the preceding one. It seemed, however, as if the relation between the relative number r and the spot area Δ were dependent on Δ . Therefore I introduced lastly

$$\Delta = k \cdot r^n$$

where the constants k and n were determined. The results of the numerical computations in these three cases were as follows:

$$\begin{aligned}\Delta_1 &= 15.98 \cdot r, \\ \Delta_2 &= -65 + 17.23 \cdot r, \\ \Delta_3 &= 7.466 \cdot r^{1.185}.\end{aligned}$$

In order to get an idea of the accuracy of these three formulæ, I calculated the mean deviations ϑ between the computed and observed values of Δ and found

$$\begin{aligned}\vartheta_1 &= 64.3, \\ \vartheta_2 &= 54.4, \\ \vartheta_3 &= 48.5.\end{aligned}$$

A computation of the percentage of mean deviations in the three cases gave the values 0.27, 0.24 and 0.15. Hence it follows that of the formulæ given above the third gives the best result. In Table III, I have given the relative number r and the spot area Δ in millionths of the sun's visible disk from 1874—1910, and figure 1 gives a graphical representation of the connection between the observed values of Δ , and the values computed from the relative numbers with the help of the formula

$$\Delta = 7.466 \cdot r^{1.185}.$$

The figure gives an unexpectedly close agreement between the observed and computed values, which is the more surprising since these relative numbers are rather arbitrarily chosen.

After this digression we return to our historical summary.

By a very diligent collection of old sun-spot observations ever since 1610, WOLF succeeded in fixing the period indicated by SCHWABE at about 10 years. In the first discussion of this extensive material WOLF got a period of

$$11.111 \pm 0.038 \text{ years}^1.$$

In this research WOLF had not introduced his relative numbers for the old observations. This was done in the year 1858. For his own observations these numbers

¹ R. WOLF: Neue Untersuchungen über die Periode der Sonnenflecken und ihre Bedeutung. Bern 1852.

were introduced in 1851. As the observations of different observers after the year 1749 partially covered each other, it was possible for WOLF to get the observations from this year converted into homogeneous relative numbers¹.

The material collected by WOLF has been employed by different authors in order to determine the period of the sun-spot activity. WOLFER, by a critical examination of the observations from 1749—1901, found the following values:²

Period from sun-spot minima = 11.141 ± 0.036 years (weight 2)

» » » maxima = 11.091 ± 0.053 » (» 1)

Mean = 11.124 ± 0.030 » .

Time from min. to max. = 5.16 years.

» » max. » min. = 5.96 »

From the material of WOLF and the photographic observations after 1874, NEWCOMB³ got a period of

$$11.13 \pm 0.02 \text{ years.}$$

The interval from minimum to the following maximum he found to be 4.62 years and from max. to min. 6.51 years, values that considerably differ from those found by WOLFER.

An interesting contribution to the question of the periodicity of the spot-activity is given by SCHUSTER⁴, though many distinguished investigators doubt the results of his researches. As is well known, SCHUSTER makes use of Harmonic Analysis. He has shown that the spot-frequency may be represented by a series of sine-curves. For the most prominent of these he has also determined the periods and found them to be as follows:

1:0. 11.125 years,

2:0. 8.36 » ,

3:0. 4.8 » .

Very remarkable is the fact that we have

$$\frac{1}{3} \cdot 33.375 = 11.125,$$

$$\frac{1}{4} \cdot 33.375 = 8.344,$$

$$\frac{1}{7} \cdot 33.375 = 4.768.$$

We might therefore consider these three waves as harmonics of a wave, the period of which is somewhat longer than 33 years.

A method similar to that of SCHUSTER is also employed by the Japanese Astronomer HISASHI KIMURA⁵. His investigations of the relative numbers of WOLF

¹ R. WOLF: *Astronomische Mitteilungen* 50. 1880.

² A. WOLFER: *Astronomische Mitteilungen* 93. 1902.

³ S. NEWCOMB: On the period of the solar spots. *Astrophysical Journal* 1901. Vol. XIII.

⁴ A. SCHUSTER: *Proceedings of Royal Society* 77. 136—140. 1905. *Phil. Transactions A* 206. 69—100. 1906.

⁵ H. KIMURA: On the Harmonic Analysis of Sun-Spot Relative Numbers. *Monthly Notices*. May 1913.

have given a result of not less than 29 periods, with a length varying between 164 and 3,6 years. Have these periods an actual existence, or are they to be considered as dependent only on the method employed? Even if the spot-frequency is represented by a series of the form

$$\Sigma A \sin \left(\frac{2\pi}{T} + H \right)$$

it does not follow that the periods of the different terms are of real import. The old theory of Epicycles represented also the motions of the planets with considerable accuracy. Professor TURNER¹, in a discussion of the results of KIMURA, remarks that the periods computed by KIMURA are as 1:2:3:4:..., and that all these periods may be considered as subperiods of a longer period of 156 years. This circumstance, however, may be considered as a control of the numerical computations of KIMURA. But from this it does not follow that these periods have an actual existence. This remains to be proved, and is a problem which will not be dealt with here.

The Motions of the Sun-spots. From the motions of the spots over the disk of the sun, the rotation-period and the position of the sun's equator may be computed. In the following we suppose

i = the angle between the sun's equator and the ecliptic.

L = the longitude of the ascending node of the equator.

As above mentioned, SCHEINER had for these quantities got the values

$$i = 7^{\circ}.5,$$

$$L = 69^{\circ}.5.$$

Amongst other determinations, I mention only the following here:

$$\text{LAUGIER:} \quad \left. \begin{array}{l} i = 7^{\circ}9' \\ L = 75^{\circ}8' \end{array} \right\} 1840.0$$

$$\text{BÖHM:} \quad \left. \begin{array}{l} i = 6^{\circ}56'.7 \\ L = 76^{\circ}46'.9 \end{array} \right\} 1833.0$$

$$\text{CARRINGTON:}^2 \quad \left. \begin{array}{l} i = 7^{\circ}15' \\ L = 73^{\circ}40' \end{array} \right\} 1850.0$$

$$\text{DYSON and MAUNDER:}^3 \quad \left. \begin{array}{l} i = 7^{\circ}10'.6 \\ L = 73^{\circ}50'.6 \end{array} \right\} 1850.0$$

¹ H. TURNER: On the Harmonic Analysis of WOLF's Sunspot Numbers, with special Reference to Mr. KIMURA's Paper. Monthly Notices. May 1913.

² R. C. CARRINGTON: Observations of the spots on the sun from Nov. 9, 1853 to March 24, 1861.

³ F. W. DYSON and E. W. MAUNDER: The position of the sun's axis as determined from photographs of the sun, 1874—1911, measured at the Royal Observatory, Greenwich. Monthly Notices LXXII.

Up to the year 1863, when CARRINGTON published his classic work, the prevailing idea was that the sun rotated as a rigid body, thus with the same angular velocity at all latitudes. It is true, SCHEINER and also some other investigators after him had observed that spots at different heliographic latitudes moved with different velocities, but it was CARRINGTON that first succeeded in proving that *the sun's angular velocity diminished as the latitude increased*. This fact CARRINGTON expresses in the following formula, where ξ is the daily rate of rotation, and β the solar latitude:

$$\xi = 865' - 165' \sin^2 \beta.$$

SPÖRER¹, from his first observations at *Anclam* 1861—1871, which have ever since been constantly continued, has verified the result of CARRINGTON. After trying different expressions for the rotation of the sun, he accepts the following, which he found to represent the observations in the best way:

$$\xi = 512'.9 + 347'.9 \cos \beta.$$

In an interesting paper of 1871, ZÖLLNER², from certain theoretical assumptions, comes to a law of rotation of the form of

$$\xi = \frac{M - N \sin^2 \beta}{\cos \beta}.$$

From the observations of CARRINGTON, he gets

$$\xi = \frac{863'.4 - 620' \sin^2 \beta}{\cos \beta}.$$

An extrapolation of these formulæ, outside the dominion of the observations, is naturally not allowed.

The most complete examination of the solar rotation, deduced from the motions of the spots, is that made by Mr and Mrs MAUNDER³ in 1905. As the material employed by the MAUNDERS is also the basis of the researches of the present paper, I shall in the next chapter give a more complete account of these observations. In the determination of the rotation-period, all such spots or groups of spots are excluded as have not been observed for at least *six consecutive days*, whereas the mean of the positions of the group on the first three days was taken as the first position of the group; the mean of the positions on the last three days as the second position. From these two positions the daily drift in longitude, referred to a system of co-ordinates, rotating in 25.38 days, was computed. From the drifts of the spots, referred to this system of co-ordinates, the mean rotation-period at different latitudes was obtained.

¹ G. SPÖRER: Beobachtungen der Sonnenflecken zu Anclam. Publikationen der Astronomischen Gesellschaft XIII. 1874.

² F. ZÖLLNER: Über das Rotations-Gesetz der Sonne und der grossen Planeten. Astronomische Nachrichten 1872. No 1849--1852.

³ W. MAUNDER and A. S. D. MAUNDER: The solar rotation period from Greenwich Sunspot Measures 1879—1901. Monthly Notices LXV.

The process employed by the MAUNDERS is very convenient when only averages of velocities are to be computed. For reasons that will be discussed later on, I have preferred to compute a daily drift of each spot from the co-ordinates on *two consecutive days*.

The MAUNDERS have also determined the solar rotation-period from recurrent spots.

From the investigation now discussed, it appears with all desirable clearness that »the differences of rotation-period of the various groups in any particular zone of latitude are much greater than the differences of mean rotation between the different zones». This fact has, it is true, been observed before, but from the correlation table between synodic rotation-period and latitude given by the MAUNDERS it has been clearly proved. From this we see that the spots are subject to considerable proper motions in longitude as well as in latitude, which fact has first been shown by CARRINGTON. Whether the distribution of the motions is subject to any law, or whether it only follows the usual GAUSSIAN law of errors, does not appear from the paper of the MAUNDERS. CARRINGTON, it is true, declares that the latitude motions of the spots are distributed in such a way that spots situated more than 20 degrees from the equator, are upon the whole, moving towards the poles, and that the motions between 10 and 20 degrees are, in general, directed towards the equator. The mean of the motions towards the poles is about 2' a day, whereas the motions towards the equator only amount to half. As far as I know, this statement has never since been verified.

A couple of the more interesting results which the MAUNDERS have obtained in their paper, are the following:

»The recurrent spots give a somewhat longer period in the mean, and are more accordant inter se than are the groups treated separately.»

»The curve given by the different rotation-periods, is not precisely symmetrical with respect to the equator, the zone of shortest rotation being north of the equator»

For the daily rate of rotation the MAUNDERS give the expression

$$\xi = 866'.6 - 128 \sin^2\beta,$$

computed from recurrent spots.

Using the material the MAUNDERS have brought together in their correlation-table between rotation-period and heliographic latitude, HIRAYAMA¹ has obtained some interesting results, which in a later chapter will be further discussed. He finishes his work with the following words:

»The present investigation, though cursory, leads me to conclude that there are two apparent drifts in the motions of the sun-spots. The angular velocity of drift I is represented by

$$\xi = 14^0.37 - 2^0.97 \sin^2\beta,$$

¹ S. HIRAYAMA: The systematic motions of sun-spots. Journal of the College of Science, Imperial University of Tokyo Vol. XXXII, 1912.

and that of drift II by

$$\xi = 14^{\circ}.69 - 2^{\circ}.65 \sin^2 \beta.$$

The mean ratio of the number of sun-spots in maximum II to those in maximum I is 1:2. — — — I do not claim that the conclusion I have arrived at, should do more than approximate to quantitative precision.»

Hitherto we have principally given an account of the motions in longitude. Concerning the latitude-motions of the spots, it seems still to be uncertain whether systematic motions do exist. CARRINGTON's assumption of two streams directed towards the poles and the equator does not seem to have been confirmed by later investigations. TURNER has, in some papers, dealt with the problem of the latitude-drift of the sun-spots, in connection with a more accurate determination of the rotation-axis of the sun. It is easily seen that an apparent latitude-drift with a period of one year may be explained by the assumption of a wrong determination of this axis. From the researches of TURNER, no conclusion can be drawn as to whether a real latitude-drift exists, or whether the axis of the sun needs a correction. Another alternative is that the sun's equator might be subject to a precessional motion, though this seems very improbable, the sun not being visibly flattened.

DYSON and MAUNDER¹ have recently carefully examined the question of the position of the rotation-axis. They call attention to the fact that changes in the apparent latitude of a spot-group may arise from three causes:

- 1:o. An error in the assumed position of the sun's axis (CARRINGTON's values $I = 7^{\circ}15'$; $L = 73^{\circ}40'$. 1850.0).
- 2:o. An actual motion of the spot on the sun's surface.
- 3:o. Errors of observations or of measurement; particularly such as would introduce error in the determinations of the photographs.

To these three causes one might also add the influence of the refraction in the solar atmosphere.

Should an error occur in the position of the assumed rotation-axis, it would cause an *apparent* drift in the latitude-motions of the spots, even if there existed no *actual* drifts. DYSON and MAUNDER, however, in their researches have not found such a period of one year in the motions of the spots. But taking into consideration the great mean errors in the averages, that is not so strange. As the result of this investigation the authors get

$$\begin{aligned}\Delta I &= -4'.4; \\ \Delta L \sin I &= +1'.7.\end{aligned}$$

Here ΔI designates the correction of the assumed inclination I , and ΔL the correction of the longitude of the ascending node of the sun's equator. Regarding the actual

¹ F. W. DYSON and E. W. MAUNDER: The position of the sun's axis as determined from Photographs of the sun from 1874—1911, measured at the Royal Observatory, Greenwich. Monthly Notices, May 1912.

latitude-drift of the spots, it seems as if there existed a northerly drift, a little greater on the northern than on the southern hemisphere. But as no mean errors are computed, it is difficult to say whether this difference is merely accidental.

In the discussion of the position of the rotation-axis in connection with the latitude-motions of the spots, we recur to the writings of DYSON and MAUNDER.

The Distribution of the Sun-spots. Concerning the distribution of the sun-spots on the sun's surface, the investigations have up till now almost exclusively dealt with the distribution in latitude. Even SCHEINER had observed that there never occurred any spots in the neighbourhood of the poles of the sun, but only in two zones on both sides of the equator. This point was not perfectly clear until the nineteenth century, when the extensive material of observation rendered it possible to examine the distribution completely. Then it became evident that sun-spots at higher latitudes than 35° were only sporadic phenomena. Very seldom does any spot occur at higher latitudes than 40° . Within the sun-spot belt, 80° wide, the spots occur mainly in two zones in either hemisphere, between the latitudes 5° and 20° .

As SPÖRER¹, from his observations between 1854—1880 has shown, the mean heliographic latitude of the sun-spots is subject to regular variations. The first sun-spots of every new period appear on higher latitudes, and as the period advances, the mean latitude of the spots is diminished in both hemispheres. This peculiarity, known as »the law of SPÖRER» has since been further confirmed by MAUNDER². The researches by MAUNDER embrace the time from 1874—1902, and are founded on the photographs of the sun taken at *Greenwich* and other English observatories. In table IV, I have placed those observations together, on which the present paper is based. From this table the law of SPÖRER appears with all desirable clearness.

On the distribution of the sun-spots in heliographic longitude no researches have been made hitherto, as far as I know. Several times, however, it has been observed that an intense spot-activity takes place on opposite sides of the sun at the same time. A research into the distribution of the spots in longitude is made more difficult as the sun rotates with different angular velocity at different latitudes. This circumstance makes it inconvenient to extend the research for any long period.

A comparison between the number of spots on the left and on the right halves of the sun gives the unexpected result that there are more spots observed on the left half than on the right. It is evident that from this fact we must not draw the conclusion that there actually are more spots on the left side of the central meridian than on the right, provided that the earth has not considerable influence on the spot-

¹ G. SPÖRER: Beobachtung der Sonnenflecken etc. *Astronomische Nachrichten* 96. 1880.

² E. W. MAUNDER: Note on the Distribution of Sun-Spots in Heliographic Latitude. *Monthly Notices LXIV*, 1904.

TABLE IV.
The Distribution of the Sun-spots in Latitude 1886—1909.

Latitude	86	87	88	89	90	91	92	93	94	95	96	97	98	99	00	01	02	03	04	05	06	07	08	09	Totals
+35.0 < β < -37.4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+32.5 < β < +34.9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
+30.0 < β < +32.4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	21
+27.5 < β < +29.9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	68
+25.0 < β < +27.4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	167
+22.5 < β < +24.9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	359
+20.0 < β < +22.4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	598
+17.5 < β < +19.9	13	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	872
+15.0 < β < +17.4	13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	895
+12.5 < β < +14.9	16	23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1242
+10.0 < β < +12.4	19	25	9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1336
+7.5 < β < +9.9	32	12	6	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1186
+5.0 < β < +7.4	16	11	3	10	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	711
+2.5 < β < +4.9	9	6	10	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	435
+0.0 < β < +2.4	6	11	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	153
— 0.1 > β > -2.5	27	11	10	10	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	176
— 2.6 > β > -5.0	35	27	33	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	485
— 5.1 > β > -7.5	51	29	25	25	10	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	957
— 7.6 > β > -10.0	53	51	30	27	5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1067
— 10.1 > β > -12.5	60	32	12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1169
— 12.6 > β > -15.0	44	14	13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1428
— 15.1 > β > -17.5	43	4	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1264
— 17.6 > β > -20.0	19	13	—	9	7	62	100	172	100	77	88	19	10	—	—	—	—	—	—	—	—	—	—	—	968
— 20.1 > β > -22.5	4	—	—	15	13	43	111	112	36	25	51	8	—	—	—	—	—	—	—	—	—	—	—	—	727
— 22.6 > β > -25.0	5	—	—	2	23	27	93	37	11	32	3	—	—	—	—	—	—	—	—	—	—	—	—	—	369
— 25.1 > β > -27.5	1	—	—	9	14	28	48	29	39	14	—	—	—	—	—	—	—	—	—	—	—	—	—	—	214
— 27.6 > β > -30.0	—	—	—	—	8	—	31	41	21	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	126
— 30.1 > β > -32.5	—	—	—	—	—	—	30	19	9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	77
— 32.6 > β > -35.0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	17
— 35.1 > β > -37.5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
Totals	466	271	158	118	162	754	1408	1890	1649	1261	711	545	477	236	172	44	70	447	823	1187	1125	1192	1039	895	17100

activity of the sun. STEPHANI, it is true, is of opinion that of the greater sun-spots during 1905—1910, only about 8 % arise on the side of the sun directed towards the earth. If this were true, we should also have an explanation of the observed spot-excess. The assertion of STEPHANI, however, is not in the least confirmed by the material of observation treated in the present paper. In another chapter we shall return to this question.

The two hemispheres also show at times considerable differences as regards the spot-activity. Thus no spots were observed in the northern hemisphere from 1672—1704. NEWCOMB draws attention to a growing preponderance of spots in the southern hemisphere in the four cycles 1856—1898¹. In the above mentioned paper »On the Distribution of Sun-Spots in Heliographic Latitude» MAUNDER shows, that the spotted area in the northern hemisphere amounts only to 43.5 % of the entire spot area. During the last period, however, the spot-activity of the two hemispheres has been of about equal intensity, as is seen from table VI.

I here finish the historical summary of the sun-spot research. The present investigation will deal, in the first place, with *the motions of the sun-spots*. In this connection I borrow the following figurative illustration from ABBOT's »The Sun»: »If we imagine an observer on the moon to watch the clouds on the earth's surface, they would appear to him on the whole to indicate a mean rotation period of about twenty-four hours for the earth. But he would also discover that many, and perhaps nearly all, of the cloudy areas had proper motions of their own besides, so that no single cloud would give correctly the rotation period of the earth. So it is with the sun-spots, for, after allowing for the sun's average rotation period, nearly every spot has a motion of its own.»

The purpose of this work is, in the first place, to investigate these proper motions of the sun-spots. In this research I shall, however, not confine myself only to the computing of *means* as is the case in the majority of the works on the motions of the sun-spots hitherto published. In order to obtain a complete idea of the distribution of the motions it is also necessary to compute the *dispersion*, *skewness* and *excess*.

The method employed in treating the observations, I have borrowed from Mathematical Statistics, chiefly from the works of CHARLIER.

Amongst these works, I especially mention »Researches into the Theory of Probability» and »Studies in Stellar Statistics I, II» published in the »Meddelanden från Lunds Astronomiska Observatorium».

¹ CH. ABBOT: The Sun, p. 193.

CHAPTER I.

The Material of Observation and its first Treatment.

All the material employed in this paper is published in the »Greenwich Observations» which appear yearly. The observations here published consist of measurements of solar photographs taken at *Greenwich*, *Dehra Dûn* and at the *Kodaikanal Observatory* in India, and at the *Royal Alfred Observatory of Mauritius*. In collecting the material from all these observatories it has been possible to obtain photographs of the sun almost every day during the last 30 years.

Table V shows us the number of photographs received each year. From 1873—1881 all the photographs are taken at Greenwich. Thus these years are rather incomplete. During the time from 1882—1884 the photographs are from *Greenwich* and *Dehra Dûn*, *Northwest Provinces, India*; 1885—1903 from *Greenwich*, *Dehra Dûn* and the *Royal Alfred Observatory of Mauritius*. Since the year 1904 the solar photographs are also received from the *Kodaikanal Observatory* in *Southern India*.

The Greenwich photographs are either taken with the Thompson or with the Dallmeyer Photoheliograph. The first of these instruments gives a solar image of about 7.5 inches, while the latter gives an image of nearly 8 inches on the photographic plate. At *Dehra Dûn* as well as the *Kodaikanal Observatory* Dallmeyer Photoheliographs are employed, which also give images of about 8 inches.

Owing to the kindness of Mr. DYSON, the Royal Astronomer, I am able to give a reproduction of one of these plates (Pl. I). A sun-spot of the size shown in this figure is naturally of very rare occurrence. Its area is thus proved to be 2771 millionths of the visible hemisphere of the sun.

TABLE V.
Days with Solar Photographs.

Year	Days with Photo- graphs	Year	Days with Photo- graphs
1873	58	1892	362
1874	152	1893	360
1875	161	1894	364
1876	162	1895	364
1877	168	1896	364
1878	146	1897	364
1879	126	1898	363
1880	158	1899	364
1881	181	1900	360
1882	343	1901	359
1883	340	1902	349
1884	315	1903	350
1885	354	1904	363
1886	363	1905	364
1887	361	1906	364
1888	358	1907	365
1889	360	1908	366
1890	361	1909	364
1891	363	1910	365

As there is given a complete description of the measures of the photographs in every volume of the Greenwich Observations, I need not enter into any discussion of these measures. The results of the measures are published in two different ways. As I shall in the following only employ the »Ledgers» where the the measures are given in a more condensed form, I will here confine myself to giving a description of these. In table VI I give an extract from the »Ledgers». According to the description given in the Greenwich Observations »the *first* column gives for each day on which the group was observed, the Greenwich civil time at which each photograph was taken, expressed by the day of the month (civil reckoning) and the decimals of a day reckoning from Greenwich midnight. The *second* and *third* columns give the sums for each day of the projected areas of all the umbræ and whole spots comprised in the group, the projected area being the area as it is

TABLE VI.
Extract from the Ledgers of Groups of Sun-spots. Group 1833.

Date Greenwich Civil Time	Projected Area of		Area for Group		Mean Longitude of Group.	Mean Latitude of Group.	Longitude from Central Meridian
	Umbræ	Whole Spot	Umbræ	Whole Spot			
1886							
Jan. 30.263	0	110	0	196	106°.3	— 14°.7	— 74°.7
31.231	20	159	21	167	106°.1	— 14°.8	— 62°.0
Febr. 1.480	32	254	23	182	106°.1	— 15°.0	— 45°.6
2.289	67	403	41	246	107°.1	— 15°.6	— 34°.0
3.280	81	403	44	219	107°.4	— 15°.0	— 20°.6
4.567	45	419	24	213	108°.4	— 15°.4	— 2°.7
5.148	39	360	23	209	109°.4	— 15°.3	+ 6°.0
6.479	30	201	16	109	108°.3	— 14°.6	+ 22°.4
7.239	15	172	9	102	107°.5	— 15°.1	+ 31°.7
8.459	22	101	16	78	109°.7	— 14°.9	+ 49°.8
9.449	9	51	10	68	113°.8	— 13°.5	+ 67°.0
10.150	0	10	0	13	106°.2	— 15°.0	+ 68°.5
11.151	0	7	0	18	104°.5	— 14°.9	+ 80°.0
Means	—	—	17	140	107°.75	— 14°.91	—

measured upon the photograph, uncorrected for foreshortening, and expressed in millionths of the sun's apparent disk. The *fourth* and *fifth* columns give the sums for each day of the areas of all the umbræ and whole spots comprised in the group, corrected for foreshortening, and expressed in millionths of the sun's visible hemisphere. The *sixth* and *seventh* columns give the mean longitude and latitude of the group, found by multiplying the longitude and latitude of each separately measured component of the group by its area, and dividing the sum of the products by the sum of the areas. The last column gives the mean longitude of the group from the central meridian, and is found by subtracting the longitude of the centre of the disk from the mean longitude of the group. At the foot of these daily results for each group are given the mean areas of umbræ and whole spots and the mean longitude and latitude for the period of observation.»

Here I only want to define the meaning of the longitude and latitude of a spot. These co-ordinates are referred to a system with the origin in the centre of the sun. The fundamental plan coincides with the equator determined by CARRINGTON, the inclination of which to the ecliptic is $7^{\circ}15'$. The longitude of the ascending node is $73^{\circ}40'$. This system of co-ordinates is assumed to rotate with the mean angular velocity of the sun as determined by CARRINGTON, corresponding to a rotation period of 25.38 days. As zero-meridian in this system that meridian is assumed which passed through the ascending node of the sun's equator on the ecliptic at Greenwich Mean Noon January 1 1854.

As regards the computation of the heliographic longitude and latitude of the spots, I refer to the Greenwich Observations, where complete formulæ are given.

TABLE VII.
Daily Mean Areas of Sun-spots

Year	Northern Hemi- sphere	Southern Hemi- sphere	Year	Northern Hemi- sphere	Southern Hemi- sphere
1874	275	362	1893	517	941
1875	153	119	1894	543	739
1876	32	90	1895	565	409
1877	26	65	1896	203	340
1878	24	1	1897	196	318
1879	14	35	1898	110	266
1880	257	159	1899	23	88
1881	500	229	1900	26	49
1882	443	558	1901	22	7
1883	340	815	1902	42	21
1884	478	601	1903	132	208
1885	283	528	1904	268	220
1886	76	305	1905	750	440
1887	44	134	1906	539	239
1888	20	69	1907	488	593
1889	5	73	1908	316	381
1890	53	46	1909	299	393
1891	401	169	1910	66	198
1892	607	607			

in one of the periods but not found in the other. Another reason why I began with the year 1886 was that at the beginning of the work I had not the Ledgers of the preceding years at hand. Further, from the year 1886 (or 1885) there are photographs of the sun for almost every day. And lastly, the two periods 1886—1897 and 1898—1909 correspond to about two cycles in the spot-activity of the sun, as is seen from table VII, where the daily mean areas denote the mean of the areas of the spots, corrected for foreshortening and expressed in millionths of the sun's visible hemisphere. From this table we also see a considerable difference between the spot-activity of the northern and southern hemispheres.

As the principal object of the present paper is to examine whether any physical drifts exist in the motions of the sun-spots, it seems convenient to treat the observations in a different way from that of other authors who have dealt with the motions of

The following investigation comprises the observations from 1886—1909. These 24 years I have divided into two periods, 1886—1897 and 1898—1909. The reasons why I began the investigation with the year 1886 are rather arbitrary. It seemed convenient to include as many years as would make it possible to divide the material into two periods in treating it. Thus one might compare the results from the two periods and be prevented from drawing conclusions from any regularities that are indicated

the sun-spots. In investigations of the rotation period at different latitudes, the writers have either limited the research to recurrent spots, or to such spots as have at least been observed during, say, six consecutive days.

This method of treatment is naturally sufficient if only averages are to be computed. But if the aim is to get a measure of the disturbing forces or currents in the solar atmosphere this method does not seem sufficient. I have therefore at last resolved to follow each spot day by day. Thus, all spots are included in the first treatment that have been observed on the photographs for at least two consecutive days. There is, however, one limitation. Since the co-ordinates of the spots are not so accurately determined in the neighbourhood of the limb of the sun, I have excluded all spots situated more than 60° from the central meridian. It is true that a great number of observations are thus excluded which might be very useful, especially in an investigation of the refraction in the solar atmosphere, but the observations are nevertheless very numerous.

Before any investigation of the motions could be made, it was necessary to transfer the contents of the Ledgers into a card-catalogue. In this catalogue I have written on each card those measures that might be useful in the investigation, as the number of the spot-groups, the time when the photograph was taken, the longitude and latitude of the spot referred to the above-mentioned system of co-ordinates, rotating in 25.38 days, the distance of the spot or of the spot-group from the central meridian, the area of umbra and of the whole spot, corrected for foreshortening and expressed in millionths of the visible hemisphere. I have, moreover, written on the cards the motions of the spot in longitude and latitude referred to the assumed system of co-ordinates, expressed in minutes of arc per hour. Lastly I have introduced the logarithm of the spot-area for reasons that will be given in another place.

In order to obtain the motions of the spots in longitude and latitude it was necessary to compute the differences between the longitudes and latitudes on two consecutive days, as well as the difference between the times when the photographs were taken. The differences between the co-ordinates, given in degrees, were afterwards reduced to minutes of arc per hour. This preparatory work alone required a considerable time. Thus it was necessary to compute about 55,000 differences and to do 18,000 multiplications in order to obtain these motions in longitude and latitude.

As the longitudes and latitudes are given with an accuracy of 0.1 and the motions are nevertheless computed with an accuracy of 0.1 per hour, we see from this fact that the motions are given with double accuracy. Supposing however that possibly existing errors in the longitudes and latitudes form a normal distribution, the errors in the motions also follow the usual GAUSSIAN law.

As an example of the cataloguing of the Ledgers we choose the spot-group No. 1833 (Table VI). As the spot during the first two and the last three days has a longitude from the central meridian greater than 60° , we omit these five

days. During eight days the spot is within 60° from the central meridian. The co-ordinates etc. during these eight days, we write on seven cards only, as we also need for each card the difference between the longitudes and latitudes on two consecutive days.

The quantities written on the cards I designate as follows:

$$\begin{aligned}\lambda &= \text{the longitude of the spot,} \\ \beta &= \text{» latitude » »} \\ x &= \Delta\lambda \cos \beta = \text{the motion in longitude of the spot multiplied by } \cos \beta, \\ y &= \Delta\beta = \text{» » » latitude » » »} \\ l &= \text{the distance from the central meridian,} \\ \Delta &= \text{the area of the whole spot expressed in millionths of the sun's} \\ &\quad \text{visible hemisphere,} \\ \omega &= \log \Delta.\end{aligned}$$

When the observations have been transferred into a catalogue of this kind, it is possible to classify the material from different points of view without too much work.

CHAPTER II.

The Longitude Motions of the Sun-spots.

A. The Classification of the Observations.

In order to be able to examine the motions in longitude as well as in latitude the whole material must first be divided into classes according to $\Delta\lambda \cos \beta$ and $\Delta\beta$. (For the meaning of this notation see page 20).

This classification was done in the following manner. The whole material of observation was first classified according to years. In each year the cards were afterwards distributed according to the latitudes of the spots, so that all the spots the latitudes of which satisfied the relation

$$0^{\circ}.0 \leq \beta \leq + 2^{\circ}.4$$

were taken together, and in the same way all those where

$$+ 2^{\circ}.5 \leq \beta \leq + 4^{\circ}.9 \text{ etc.}$$

For the southern hemisphere the limits of the classes were determined from

$$- 0^{\circ}.1 \geq \beta \geq - 2^{\circ}.5 \text{ etc.}$$

The number of cards in each such class will be seen in table IV.

To begin with I was uncertain whether it would be necessary to divide the material into latitude-zones as narrow as $2^{\circ}.5$. I fixed, however, on this class-interval. Should it prove during the progress of the work that a class-interval of $5^{\circ}.0$ was sufficient it would then be an easy matter to conjoin the classes.

When this classification into latitude-zones of all the years comprised in the investigation had been performed we could proceed to arrange the material in each such zone according to the motions in longitude and latitude, i. e. according to $\Delta\lambda \cos \beta$ and $\Delta\beta$. In the classifying of the material according to the size of $\Delta\lambda \cos \beta$ I fixed on a class-interval of $1^{\circ}.0$. As it proved that the motions in latitude amounted to about half this value, I chose the class-interval for this classification at $0^{\circ}.5$. The class where

$$- 1^{\circ}.0 \leq \Delta\lambda \cos \beta \leq - 0^{\circ}.1$$

was indicated by 0.

The class in the classification according to $\Delta\beta$, where

$$-0'.5 \leq \Delta\beta \leq -0'.1$$

received the same number.

The result of this division according to $\Delta\lambda \cos \beta$ and $\Delta\beta$ is given in the form of correlation-tables.

It happened very seldom that spots occurred for which these motions amounted

TABLE VIII.
The Number of Observations in
different Years.

Year	All Observa- tions	Excluded Observa- tions	Remaining Observa- tions
1886	466	12	454
1887	271	3	268
1888	158	4	154
1889	118	—	118
1890	162	8	154
1891	754	15	739
1892	1408	34	1374
1893	1890	30	1860
1894	1649	32	1617
1895	1261	13	1248
1896	711	16	695
1897	545	4	541
1898	477	9	468
1899	236	2	234
1900	172	1	171
1901	44	—	44
1902	70	1	69
1903	447	8	439
1904	823	24	799
1905	1187	50	1137
1906	1125	35	1090
1907	1192	22	1170
1908	1039	24	1015
1909	895	11	884
Totals	17100	358	16742

to more than $10'$ per hour in longitude and $5'$ in latitude. After much hesitation, I have decided to exclude all such observations in making the correlation-tables and in the computation of the characteristics of the motion (*mean, dispersion, skewness and excess*).

In general, very great care must be taken in excluding observations.

In a series of observed values of the same variable the probability that an observation is farther from the mean than 3σ (σ = the dispersion) is about 0.002. The observations I have omitted are, however, at a distance from the mean of about 4.505 and

TABLE IX.
The Number of Observations at different
Latitudes.

Latitude	All Observa- tions	Excluded Observa- tions	Remaining Observa- tions
+ 32.5 < β < + 34.9	10	—	10
+ 30.0 < β < + 32.4	21	—	21
+ 27.5 < β < + 29.9	68	4	64
+ 25.0 < β < + 27.4	167	7	160
+ 22.5 < β < + 24.9	359	7	352
+ 20.0 < β < + 22.4	598	15	583
+ 17.5 < β < + 19.9	872	21	851
+ 15.0 < β < + 17.4	895	21	874
+ 12.5 < β < + 14.9	1242	26	1216
+ 10.0 < β < + 12.4	1336	28	1308
+ 7.5 < β < + 9.9	1186	21	1165
+ 5.0 < β < + 7.4	711	28	683
+ 2.5 < β < + 4.9	435	7	428
+ 0.0 < β < + 2.4	153	—	153
— 0.1 > β > — 2.5	176	1	175
— 2.6 > β > — 5.0	485	5	480
— 5.1 > β > — 7.5	957	15	942
— 7.6 > β > — 10.0	1067	13	1054
— 10.1 > β > — 12.5	1169	25	1144
— 12.6 > β > — 15.0	1428	24	1404
— 15.1 > β > — 17.5	1264	19	1245
— 17.6 > β > — 20.0	968	23	945
— 20.1 > β > — 22.5	727	16	711
— 22.6 > β > — 25.0	369	14	355
— 25.1 > β > — 27.5	214	12	202
— 27.6 > β > — 30.0	126	4	122
— 30.1 > β > — 32.5	77	2	75
— 32.6 > β > — 35.0	17	—	17
— 35.1 > β > — 37.5	3	—	3
Totals	17100	358	16742

more. In a normal distribution the probability of such a deviation is approximately 0.00001. There is moreover another thing. Several of these great motions may be thus explained, that in a spot-group, consisting of a long line of spots, part of these spots disappear, which may evidently cause a displacement of the centre of gravity of the spot.

As the co-ordinates of the group are co-ordinates of the centre of gravity, it is evident that great proper motions, caused in this way, are only apparent, which fact justifies the exclusion of such great motions. Since it is here a question of giving the essential features of the state of the motions of the spots, it seems the more necessary to exclude these motions from the investigation as they would have a very considerable influence especially on the higher characteristics, skewness and excess.

Tables VIII and IX show the distribution of the cards (or observations) according to years and latitudes before and after the exclusion.

In table X, showing the distribution of the motions in longitude, I have given an extract from the tables of correlation, without taking into account the corresponding motions in latitude.

B. Characteristics of the Motions in Longitude.

Summary of the Method employed. Before proceeding to a computation of the characteristics of the motions of the spots, I will give a summary of the method employed in this calculation. This method has been developed by Prof. CHARLIER¹ in several papers.

For short, we put in the following

$$x = \Delta\lambda \cos \beta$$

and

$$x_0 = \text{the mean of } x.$$

Suppose $F(x)$ to be the frequency-function of x . As this function (see table X) is of type A according to the nomenclature of CHARLIER, $F(x)$ may be represented by the series

$$F(x) = A_0 \varphi(x) + A_3 \varphi^{\text{III}}(x) + A_4 \varphi^{\text{IV}}(x) + \dots$$

where $\varphi(x)$ represents the function of normal distribution

$$(1) \quad \varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}.$$

¹ For example see CHARLIER: Researches into the Theory of Probability. Lund 1906.

The above equation may, however, be written in a form more convenient for numerical purposes. We observe that

$$\varphi^s(x) = R_s(x) \varphi(x)$$

where $R_s(x)$ is a whole rational function of x of the degree s . Thus, we have

$$\begin{aligned}\sigma^2 R_1 &= -(x - x_0), \\ \sigma^4 R_2 &= +(x - x_0)^2 - \sigma^2, \\ \sigma^6 R_3 &= -(x - x_0)^3 + 3\sigma^2(x - x_0), \\ \sigma^8 R_4 &= +(x - x_0)^4 - 6\sigma^2(x - x_0)^2 + 3\sigma^4, \\ &\dots\dots\dots\end{aligned}$$

From these and preceding expressions it follows that

$$\sigma\varphi(x), \quad \sigma^4\varphi^{\text{III}}(x), \quad \sigma^5\varphi^{\text{IV}}(x), \dots\dots$$

are functions only of the variable $\frac{x - x_0}{\sigma}$.

On account of this fact we may conveniently write the expression for $F(x)$ in the form

$$F(x) = \frac{N}{5\sigma} [\sigma\varphi(x) + \beta_3\sigma^4\varphi^{\text{III}}(x) + \beta_4\sigma^5\varphi^{\text{IV}}(x) + \dots\dots].$$

or

$$(2) \quad F(x) = \frac{N}{5\sigma} [\varphi_0(x) + \beta_3\varphi_3(x) + \beta_4\varphi_4(x) + \dots\dots]$$

In this equation N designates the number of observations. The deviation from the normal distribution is indicated by β_3 and β_4 . By a distribution according to the usual law of errors, β_3 and β_4 vanish. The arbitrary factor 5 is attached in order to give the frequency-curve a suitable form. (See page 32).

The characteristics we have to compute are the following:

$$(3) \quad \left\{ \begin{array}{l} x_0 = \text{mean}, \\ \sigma = \text{dispersion}, \\ S = 3\beta_3 = \text{skewness}, \\ E = 3\beta_4 = \text{excess}. \end{array} \right.$$

These characteristics may be computed in a simple manner from the moments of the function $F(x)$. I designate the moment of the s^{th} order in reference to the origin by μ'_s . This quantity is determined by the formula

$$(4) \quad \mu'_s = \int_{-\infty}^{+\infty} F(x)x^s dx.$$

The corresponding relative moment ν'_s is obtained from

$$(5) \quad \nu'_s = \frac{\mu'_s}{N}.$$

TABLE X.
The Motions in Longitude of the Spots at different Latitudes.

Northern Hemisphere. First Period. 1886—1897.

Class-interval in $\Delta\lambda \cos \beta = 1'.0$. Provisory Mean = $-0'.5$.

Latitude	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
$0.0 < \beta < +2.4$	—	—	—	1	1	2	2	4	3	8	25	24	15	11	3	3	2	3	—	—	107
$+2.5 < \beta < +4.9$	—	—	—	—	2	1	4	10	7	23	55	42	29	13	7	3	3	5	2	1	207
$+5.0 < \beta < +7.4$	1	1	3	4	3	4	3	7	12	28	60	43	32	21	14	3	5	1	—	2	247
$+7.5 < \beta < +9.9$	—	3	1	1	4	5	10	16	24	62	114	88	59	23	24	9	6	7	3	3	462
$+10.0 < \beta < +12.4$	1	1	2	—	2	6	6	17	39	109	178	139	62	44	22	14	12	7	1	—	662
$+12.5 < \beta < +14.9$	—	—	3	2	1	8	8	13	49	113	164	124	46	36	26	12	10	7	—	1	623
$+15.0 < \beta < +17.4$	—	3	3	2	5	8	7	19	43	97	137	103	42	22	17	13	5	6	6	1	539
$+17.5 < \beta < +19.9$	—	—	1	3	5	5	17	23	53	105	138	83	34	33	19	11	2	7	3	—	542
$+20.0 < \beta < +22.4$	—	—	1	—	1	3	9	22	40	75	64	45	27	22	9	6	8	2	2	—	336
$+22.5 < \beta < +24.9$	—	—	—	2	2	7	7	17	31	51	39	27	18	8	6	4	5	1	1	—	226
$+25.0 < \beta < +27.4$	1	—	—	2	1	1	5	2	15	29	23	19	12	3	3	2	—	2	—	2	122
$+27.5 < \beta < +29.9$	—	—	—	—	2	2	1	5	12	11	13	4	1	—	1	1	1	—	—	—	54
$+30.0 < \beta < +32.4$	—	—	—	—	—	—	—	2	4	—	1	1	—	1	—	—	1	—	—	—	10
$+32.5 < \beta < +34.9$	—	—	—	—	—	—	1	1	5	2	—	—	—	—	—	—	—	—	1	—	10
Totals	3	8	14	17	29	52	80	158	337	713	1011	742	377	237	151	81	60	48	19	10	4147

Southern Hemisphere. First Period. 1886—1897.

Class-interval in $\Delta\lambda \cos \beta = 1'.0$. Provisory Mean = $-0'.5$.

Latitude	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
$-0.1 > \beta > -2.5$	1	2	—	—	—	1	1	8	3	11	25	32	12	5	3	4	2	1	2	—	113
$-2.6 > \beta > -5.0$	—	1	1	—	1	2	2	7	10	22	85	72	26	16	14	3	8	5	2	1	278
$-5.1 > \beta > -7.5$	—	—	1	2	—	5	7	18	21	81	163	95	52	41	18	5	3	2	5	2	521
$-7.6 > \beta > -10.0$	1	—	2	1	4	7	10	8	37	82	168	110	66	46	19	13	7	2	3	1	587
$-10.1 > \beta > -12.5$	1	1	—	6	5	4	17	22	69	100	163	128	71	41	23	14	8	4	2	1	680
$-12.6 > \beta > -15.0$	1	1	3	2	7	8	18	17	55	114	164	137	72	37	19	13	10	6	2	1	687
$-15.1 > \beta > -17.5$	1	2	—	4	7	5	15	18	70	118	162	96	46	32	22	16	6	3	2	—	625
$-17.6 > \beta > -20.0$	—	2	1	1	3	13	15	31	59	115	155	95	57	23	12	12	8	5	1	1	609
$-20.1 > \beta > -22.5$	—	—	1	5	5	5	6	24	48	80	91	48	38	20	7	9	8	3	2	—	400
$-22.6 > \beta > -25.0$	1	1	2	4	1	5	11	14	30	38	49	27	17	9	5	4	1	2	—	—	221
$-25.1 > \beta > -27.5$	—	1	—	—	4	4	8	18	20	31	44	18	9	6	5	3	—	—	—	1	172
$-27.6 > \beta > -30.0$	—	1	—	2	—	2	2	5	19	24	13	16	10	4	1	3	—	—	—	—	102
$-30.1 > \beta > -32.5$	—	—	—	—	1	2	3	7	15	14	8	2	2	3	1	1	1	—	—	—	60
$-32.6 > \beta > -35.0$	—	—	—	—	—	—	1	5	6	2	—	1	1	—	—	—	1	—	—	—	17
$-35.1 > \beta > -37.5$	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	3
$-37.6 > \beta > -40.0$	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
$-40.1 > \beta > -42.5$	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	6	12	12	27	38	63	116	202	463	832	1290	877	479	283	149	101	64	33	21	8	5076

TABLE X. Cont.

The Motions in Longitude of the Spots at different Latitudes.*Northern Hemisphere. Second Period. 1898—1909.*Class-interval in $\Delta\lambda \cos \beta = 1'.0$. Provisory Mean = $-0'.5$.

Latitude	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
$0.0 < \beta < +2.4$	—	—	—	—	—	—	—	1	4	4	14	10	6	3	2	1	1	—	—	—	46
$+2.5 < \beta < +4.9$	—	—	—	1	3	6	3	7	14	26	59	45	25	14	9	7	—	1	1	—	221
$+5.0 < \beta < +7.4$	1	—	1	—	1	3	10	16	22	72	119	78	52	20	17	8	12	2	1	1	436
$+7.5 < \beta < +9.9$	1	—	1	3	2	12	14	35	61	92	180	136	76	41	26	9	6	4	3	1	703
$+10.0 < \beta < +12.4$	1	—	1	2	5	7	8	38	64	117	144	112	68	29	28	12	3	4	3	—	646
$+12.5 < \beta < +14.9$	2	1	1	7	6	11	14	28	62	102	127	108	55	30	8	10	7	6	6	2	593
$+15.0 < \beta < +17.4$	—	—	3	6	5	9	4	20	36	56	70	46	27	23	9	13	3	3	1	1	335
$+17.5 < \beta < +19.9$	1	1	—	3	1	7	10	20	44	61	63	39	25	14	11	2	4	2	1	—	309
$+20.0 < \beta < +22.4$	—	2	—	1	6	3	6	10	27	62	61	29	14	6	4	5	5	3	2	1	247
$+22.5 < \beta < +24.9$	—	—	—	—	1	1	6	8	18	27	32	15	6	6	2	3	1	—	—	—	126
$+25.0 < \beta < +27.4$	—	—	—	1	—	1	—	3	8	9	9	4	1	—	1	—	—	—	1	—	38
$+27.5 < \beta < +29.9$	—	—	—	—	1	—	—	1	3	1	1	1	1	1	—	—	—	—	—	—	10
$+30.0 < \beta < +32.4$	—	—	—	—	—	—	—	1	3	—	1	2	3	—	—	—	—	1	—	—	11
Totals	6	4	7	24	31	60	75	188	366	629	880	625	359	187	117	70	42	26	19	6	3721

*Southern Hemisphere. Second Period. 1898—1909.*Class-interval in $\Delta\lambda \cos \beta = 1'.0$. Provisory Mean = $-0'.5$.

Latitude	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
$-0.1 > \beta > -2.5$	—	—	—	—	1	1	1	5	3	9	21	8	6	7	—	—	—	—	—	—	62
$-2.6 > \beta > -5.0$	—	1	—	1	1	2	4	5	11	18	66	49	21	12	4	4	—	3	—	—	202
$-5.1 > \beta > -7.5$	—	—	1	—	1	3	9	13	26	58	124	83	44	26	15	5	6	2	3	2	421
$-7.6 > \beta > -10.0$	1	—	—	5	4	4	9	18	26	55	111	95	56	38	22	12	4	4	2	1	467
$-10.1 > \beta > -12.5$	1	—	2	3	3	3	8	28	32	84	125	61	46	32	8	11	8	7	2	—	464
$-12.6 > \beta > -15.0$	—	2	6	7	9	10	12	22	69	142	174	111	70	31	20	13	9	5	4	1	717
$-15.1 > \beta > -17.5$	2	4	—	1	3	6	12	31	68	121	149	92	56	30	21	14	4	4	1	1	620
$-17.6 > \beta > -20.0$	2	1	1	2	3	7	9	17	29	66	75	51	27	21	9	9	1	5	1	—	336
$-20.1 > \beta > -22.5$	—	1	—	6	4	5	7	19	38	62	73	42	30	9	8	2	2	1	1	1	311
$-22.6 > \beta > -25.0$	—	—	—	1	1	4	3	11	18	15	29	24	12	7	3	2	3	—	1	—	134
$-25.1 > \beta > -27.5$	—	—	—	—	—	—	3	4	2	2	7	5	2	1	2	1	—	—	—	1	30
$-27.6 > \beta > -30.0$	—	—	—	—	—	—	2	1	3	8	3	1	1	—	—	—	1	—	—	—	20
$-30.1 > \beta > -32.5$	—	—	—	—	2	—	3	—	2	2	2	—	—	1	3	—	—	—	—	—	15
Totals	6	9	10	26	32	45	82	174	327	642	959	622	371	215	115	73	38	31	15	7	3799

Having computed ν_1' , ν_2' , ν_3' and ν_4' , the moments in reference to the mean are given by

$$(5) \quad \begin{cases} \nu_2 = \nu_2' - b^2, \\ \nu_3 = \nu_3' - 3b\nu_2' + 2b^3, \\ \nu_4 = \nu_4' - 4b\nu_3' + 6b^2\nu_2' - 3b^4, \end{cases}$$

in which formulæ I have written b in stead of ν_1' .

Lastly the characteristics are obtained from the relations

$$(6) \quad \begin{cases} x_0 = -0.5 + b, \\ \sigma = \sqrt{\nu_2}, \\ S = -\frac{\nu_3}{2\sigma^3}, \\ E = \frac{1}{8} \left(\frac{\nu_4}{\sigma^4} - 3 \right). \end{cases}$$

In the computation of these characteristics of the motions in longitude, the material of observation brought together in table X, is divided into latitude-zones, 5° wide, in stead of $2^\circ.5$, as is the case in the table. The results of these computations are given in table XI. The latitude-zones of the northern hemisphere are designated by N_1 , N_2 , N_3 etc. and those of the southern hemisphere by S_1 , S_2 , S_3 etc, N_1 and S_1 being nearest the equator.

Mean and Dispersion. In table XI, n represents in the first place the number of cards, *spot-days* we may say, after the manner in which the material of observation has been catalogized. The values of x_0 indicate the mean of the motions of the spots in longitude, referred to the adopted system of co-ordinates. The angular velocity of this system is $35'.461$ an hour, corresponding to a rotation period of 25.38 days. From the values of x_0 , the fact, discovered by CARRINGTON, of the angular velocity of the sun being diminished by increasing latitude, is clearly proved.

In a later section I shall compute the rotation period of different latitudes by using the values of the characteristics given in this table.

It may not be out of place to make a comment here. In the papers on the motions of the sun-spots hitherto published, there are, as far as I can remember, only the averages of the motions computed. On that account one gets no idea of the accuracy by which the rotation period, for example, may be determined from the observations of the spots. During the computations I have made in order to determine the rotation period at different latitudes from this vast material, I was greatly surprised at finding that this determination could only be made with considerable uncertainty. An excellent measure of this uncertainty is the dispersion σ_x . If we designate the mean error of x_0 by $\varepsilon(x_0)$, we have as is well known

$$(7) \quad \varepsilon(x_0) = \frac{\sigma_x}{\sqrt{n}}.$$

The adopted system of co-ordinates having an angular velocity of $35'.461$ an hour, the velocity of rotation determined from the spots of the zone N_3 during the first period would thus be

$$35'.461 + 0'.954 = 36'.415.$$

Since, according to the theory of errors, a third of the observations are outside the limits

$$x_0 \pm \sigma,$$

we have consequently about 430 of the 1285 observations $> 38'.632$ and $< 34'.198$, which fact clearly proves the uncertainty of the determinations.

TABLE XI.
Characteristics of the Motions in Longitude.

Latitude-zones	First Period (1886—1897)					Second Period (1898—1909)				
	n	x_0	σ_x	S_x	E_x	n	x_0	σ_x	S_x	E_x
N_7	20	-0.250	$+3.000$	—	—	11	$+1.545$	$+2.709$	—	—
N_6	176	$+0.108$	$+2.609$	-0.173	$+0.335$	48	-0.292	$+2.380$	—	—
N_5	562	$+0.404$	$+2.442$	-0.240	$+0.116$	373	$+0.227$	$+2.460$	-0.210	$+0.280$
N_4	1081	$+0.635$	$+2.445$	-0.092	$+0.246$	644	$+0.317$	$+2.631$	$+0.012$	$+0.150$
N_3	1285	$+0.954$	$+2.217$	$+0.060$	$+0.273$	1239	$+0.569$	$+2.447$	-0.030	$+0.249$
N_2	709	$+1.015$	$+2.668$	$+0.140$	$+0.272$	1139	$+0.835$	$+2.279$	-0.034	$+0.240$
N_1	314	$+1.296$	$+2.444$	-0.126	$+0.178$	267	$+0.934$	$+2.237$	$+0.092$	$+0.162$
S_1	391	$+1.285$	$+2.492$	$+0.082$	$+0.409$	264	$+0.792$	$+2.091$	$+0.218$	$+0.347$
S_2	1108	$+1.001$	$+2.191$	-0.062	$+0.328$	888	$+1.032$	$+2.322$	-0.031	$+0.254$
S_3	1367	$+0.754$	$+2.366$	$+0.049$	$+0.250$	1181	$+0.594$	$+2.491$	$+0.020$	$+0.246$
S_4	1234	$+0.511$	$+2.335$	-0.049	$+0.236$	956	$+0.491$	$+2.446$	$+0.095$	$+0.288$
S_5	621	$+0.246$	$+2.555$	$+0.014$	$+0.182$	445	$+0.212$	$+2.424$	-0.014	$+0.205$
S_6	274	-0.095	$+2.388$	$+0.030$	$+0.202$	50	$+0.260$	$+2.657$	—	—
S_7	77	-0.630	$+2.094$	—	—	15	0.000	$+3.386$	—	—

The values of σ_x are evidently a measure of the size of the currents in the solar atmosphere. If the sun were a rigid body with a certain rotation period and the spots were formations fixedly connected with the sun, σ_x would then be equal to zero, or still better, a measure of the accuracy with which the measurements of the co-ordinates of the spots are made. The values of σ would under such circumstances, for reasons that will be given later on, be considerably less than they are now, and only dependent on errors in the measurements of the photographs.

The great values of σ_x , given by the observations, must thus be due to an irregularity in the motions of the spots, caused by currents of some kind or other in the atmosphere of the sun.

Skewness and Excess. As regards the other two characteristics, the *skewness* S_x and the *excess* E_x , one does not usually take them into consideration in statistical researches.

The assumption $S_x = E_x = 0$, signifies that the distribution of the motions follows the usual GAUSSIAN law of errors. Experience from the applications of the

mathematical statistics to various phenomena, now shows that this accordance with the simple GAUSSIAN law must not be assumed a priori. On the contrary, the statistical analysis of different mass-appearances shows many cases in which the usual law of distribution can only be regarded as the first approximation to the actual distribution.

A glance at table XI shows that there is no pronounced skewness, whether positive or negative. In half the cases S_x is positive, in half negative. If we take into consideration the uncertainty with which S_x can be determined — the mean error in S being $1.9325 : \sqrt{n}$ — we see that, practically speaking, we might put $S_x = 0$.

I did not expect this originally. In the research made by HIRAYAMA into the systematic motions of the sun-spots, as stated in the introduction, page 11, the author shows that if the material of observation is divided according to the rotation period, a distribution is obtained, which, judging from the graphic representation of HIRAYAMA, would give a pronounced positive skewness, which should evidently correspond to a negative skewness in the distribution of the motions. But such a skewness does not exist. The discussion of this contradiction we shall relegate to another section.

Lastly, as regards the excess E_x , it is thoroughly positive, and the same circumstance is repeated in the motions in latitude. To find a plausible explanation of this seems, however, to be attended with considerable difficulties.

A positive excess may under certain circumstances arise by the addition of two or more normal distributions.

Let us take a zone with N observations of the motions in longitude. Suppose these N observations to be divided into two groups with N_1 and N_2 observations in each group. In each of these groups we assume the distribution to be normal. The dispersions we designate for a moment by σ_1 and σ_2 and the means by m_1 and m_2 . If the mean of these two groups combined is designated by m , the dispersion by σ , the skewness by S and the excess by E , the following expressions may be deduced:¹

$$(8) \left\{ \begin{aligned} m &= \frac{N_1}{N} m_1 + \frac{N_2}{N} m_2; \\ \sigma^2 &= \frac{N_1}{N} \sigma_1^2 + \frac{N_2}{N} \sigma_2^2 + \frac{N_1 N_2}{N N} (m_1 - m_2)^2; \\ 2\sigma^3 \cdot S &= \frac{3N_1 N_2}{N N} (m_1 - m_2) (\sigma_1^2 - \sigma_2^2) + \frac{N_1 N_2}{N N} \left(\frac{N_1}{N} - \frac{N_2}{N} \right) (m_1 - m_2)^3; \\ 8\sigma^4 \cdot E &= \frac{3N_1 N_2}{N N} (\sigma_1^2 - \sigma_2^2)^2 - \frac{6N_1 N_2}{N N} \left(\frac{N_1}{N} - \frac{N_2}{N} \right) (\sigma_1^2 - \sigma_2^2) (m_1 - m_2)^2 + \frac{N_1 N_2}{N N} \left(1 - \frac{6N_1 N_2}{N N} \right) (m_1 - m_2)^4. \end{aligned} \right.$$

¹ C. V. L. CHARLIER: Studies in Stellar Statistics II. page 94.

From the last of these formulæ the following appears:

$$1:o. \quad E = 0 \text{ if } m_1 = m_2; \sigma_1 = \sigma_2.$$

$$2:o. \quad E > 0 \text{ if } m_1 = m_2; \sigma_1 \neq \sigma_2.$$

$$3:o. \quad E \geq 0 \text{ if } m_1 \neq m_2; \sigma_1 = \sigma_2; 1 - \frac{6N_1N_2}{NN} \geq 0$$

or what is the same

$$E > 0, \text{ if anyone of the quotients } \frac{N_1}{N} \text{ and } \frac{N_2}{N} \text{ is } < 0.211.$$

$$E < 0, \text{ if the quotients } \frac{N_1}{N} \text{ and } \frac{N_2}{N} \text{ are within the limits } 0.211 \text{ and } 0.789.$$

Hence it follows, *that a positive excess always arises when two normal curves with the same medium and different dispersions are added.* If we should thus be able to show that our material of observation can be divided into two or more groups, with different dispersions in each group, this would possibly be a means by which one might be able to explain the positive excess.

My first idea was to make a division of the material according to the spot-area, for it seems, in all probability, that the motions of the great spots should have a smaller dispersion than the small ones, since the dispersion might also be considered as a measure of the stability of the spots during the motion.

In dividing the material according to the spot-area, it however showed that the distribution obtained was of the same type which CHARLIER in his *Researches into the Theory of Probability* calls the B-type, the number of spots diminishing with increasing spot-area. As the frequency-function of this distribution, from a mathematical point of view, is considerably more complicated than the frequency-function of the A-type, I introduced on trial $\log \Delta$ instead of Δ and divided the spots according to this new variable, which in the following I designate by ω . For those spots the areas of which are smaller than 0.5 and therefore designated by zero in the Ledgers, I have put $\Delta = 1$. Thus zero will be the lower limit of ω . It then appeared, that the distribution according to ω was almost normal.

As an example of the correlation between x and ω , I here give the table of correlation between these variables of the zone first examined (Table XII). The provisory mean of ω is 1.900 and the class-breadth is assumed at 0.200. From the form of this table, it is evident that the dispersions of the motions in longitude increase with diminishing spot-area.

A more minute investigation of the correlation between dispersion and spot-area will be the subject of another chapter. Here we will only discuss this question as far as it can serve as an explanation of the positive excess. Should the distribution of the motions of spots having a certain spot-area prove normal, we have previously shown that the distribution of all spots combined will have a positive excess, the dispersions of the motions for spots of different sizes being different. It now appear, however, that even after dividing the material according to the spot-

area Δ (or better, according to $\log \Delta$), the positive excess nevertheless remains undiminished. This may, to some extent, be owing to the fact of the class-interval in ω being too broad. But if this should be the explanation of the remaining great positive excess, this excess should have somewhat smaller values within these different groups than when all the spots are combined.

In examining the latitude motions of the spots, we shall be in a position to discuss the problem of the positive excess, by a comparison with the observations used by DYSON and MAUNDER in their paper on the position of the sun's axis.

TABLE XII.
Correlation between $\Delta \lambda \cos \beta (= x)$ and $\log \Delta (= \omega)$.

1898–1909. $-10^{\circ}.1 \geq \beta \geq -12^{\circ}.5$.

Provisory Mean in $\omega = 1.900$. Class-interval in $\omega = 0.200$.

Provisory Mean in $x = -0'.5$. Class-interval in $x = 1'.0$.

$\omega \backslash x$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+8	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	2
+7	—	—	—	—	—	—	1	—	1	1	2	3	1	—	—	—	—	—	—	—	9
+6	—	—	—	—	—	—	—	—	2	2	2	4	1	—	—	—	—	—	—	—	11
+5	—	—	—	—	—	—	—	—	3	6	12	9	—	1	—	—	—	—	—	—	31
+4	—	—	—	—	1	—	—	4	1	16	20	11	6	3	1	—	—	—	—	—	63
+3	—	—	—	—	1	2	—	2	4	14	27	14	10	2	2	—	—	—	—	—	78
+2	—	—	—	—	—	—	1	3	7	18	14	10	8	4	1	2	—	—	—	—	68
+1	—	—	—	—	—	—	2	5	8	11	18	14	8	4	3	—	1	—	—	—	74
0	—	—	—	—	—	—	2	2	8	12	16	13	9	4	3	1	—	—	—	—	70
-1	—	—	—	—	1	—	—	6	11	17	9	8	3	—	5	1	1	—	—	—	62
-2	—	—	—	—	1	2	1	1	6	8	6	8	6	3	5	4	—	2	—	—	53
-3	—	—	1	1	—	—	1	4	7	5	6	7	8	3	4	2	1	—	1	—	51
-4	—	—	—	—	1	1	—	8	2	4	5	6	3	3	4	1	—	1	—	—	39
-5	1	—	—	1	—	1	—	1	2	2	2	5	2	1	—	1	—	—	2	—	21
-6	—	—	—	—	—	1	—	2	1	1	2	—	2	1	—	—	—	1	—	—	11
-7	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	2
-8	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	1	—	1	2	5	7	8	38	64	117	144	112	68	29	28	12	3	4	3	—	646

In order to get a more accurate idea of the distribution of the longitude motions of the sun-spots, I have given in fig. 2 a graphical representation of this distribution in the zones N_3 and S_3 , compared with the normal distribution. The observations of both periods are taken together. The distribution is in the figure indicated with small circles. To be able to compare the distribution of the motions with a normal distribution, I have used the normal co-ordinates introduced by CHARLIER,

$$(9) \quad \begin{cases} X = (x - b) \frac{w}{\sigma}; \\ Y = \frac{5}{N} \frac{\sigma}{w} F(x). \end{cases}$$

In these formulæ

w = the class-interval,

$F(x)$ = the number of observations belonging to a class with a class-number x .

Only in this way is it possible to compare different distributions directly.

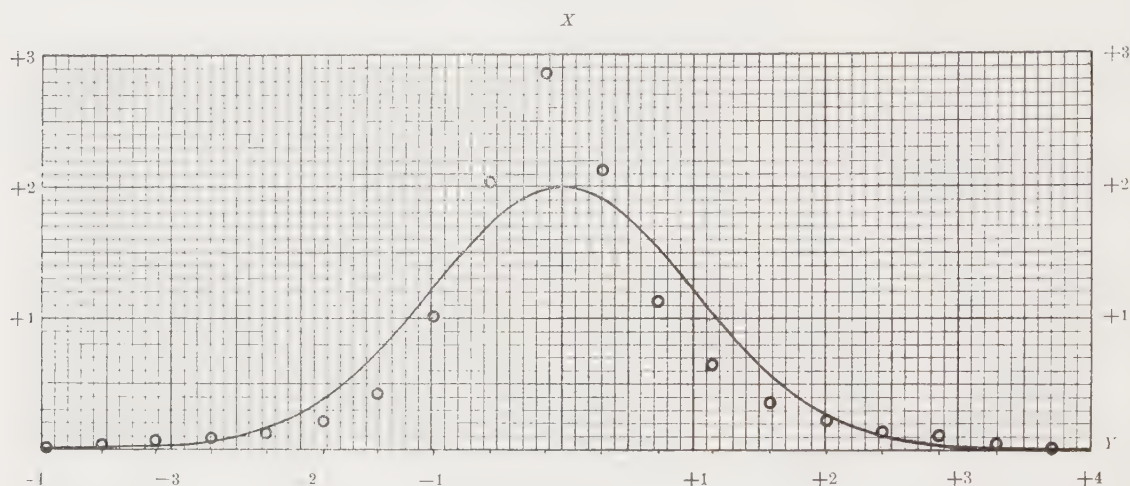


Fig. 2. The Distribution of the Motions in Longitude compared with a normal Distribution. 1886–1909: N_8 and S_8 .

In Table XIII I have given the characteristics of the motions in longitude, both hemispheres as well as both periods being taken together.

(C). The Rotation Period of the Sun, computed from the Longitude Motions of the Spots.

The longitude motions of the spots are given in reference to a system of co-ordinates the sidereal rotation period of which is 25.38 days. This rotation period we shall in the following designate by t_0 . The angular velocity v_0 is thus 35'.461 per hour. Between t_0 and v_0 the following relation exists:

$$(10) \quad t_0 = \frac{900}{v_0}.$$

In our previous study of the longitude motions of the spots we have used the quantity $\Delta\lambda \cos \beta$. In determining the rotation period we must employ $\Delta\lambda$ instead. Within each zone, 5° broad, we may regard $\cos \beta$ as a constant. In the preceding pages we have put

$$\Delta\lambda \cos \beta = x.$$

TABLE XIII.

**Characteristics of the Motions in Longitude
1886—1909.**

If we now introduce

$$\Delta\lambda = u$$

we get

$$u = x \sec \beta.$$

Putting the mean of $u = u_0$, the dispersion of $u = \sigma_u$, the skewness $= S_u$, and the excess $= E_u$, we get the following formulæ for the computation of these quantities

Zone	n	x_0	σ_x	S_x	E_x
N_1 and S_1	1236	+1.107	+2.354	+0.036	+0.305
N_2 » S_2	3844	+0.961	+2.343	+0.005	+0.284
N_3 » S_3	5072	+0.797	+2.344	+0.041	+0.298
N_4 » S_4	3915	+0.508	+2.445	-0.004	+0.236
N_5 » S_5	2001	+0.279	+2.478	-0.101	+0.190
N_6 » S_6	548	-0.015	+2.430	-0.142	+0.280
N_7 » S_7	123	-0.402	+2.667	—	—

$$(11) \quad \begin{cases} u_0 = x_0 \sec \beta; \\ \sigma_u = \sigma_x \sec \beta; \\ S_u = S_x; \\ E_u = E_x; \end{cases}$$

Thus, if u indicates the longitude motion of a spot, expressed in minutes of arc per hour, and referred to our system of co-ordinates, we then obtain the corresponding rotation period from the formula

$$(12) \quad t = \frac{900}{v_0 + u}.$$

We know the mean of u for a great number of spots and therefore also the mean of $v_0 + u$ or $M(v_0 + u)$. The question then is to compute the mean value of t or $M(t)$. In the first place we remark that

$$(13) \quad M(t) = \frac{900}{M(v_0 + u)},$$

or, expressed in words, if the mean of the motions in longitude is known, we can not, from this value alone, compute the rotation period. To do this, it is necessary also to know the dispersion, skewness and excess of the motions. Here the dispersion will have a considerably greater influence than the other two characteristics.

From the equation (12) we get

$$t = t_0 \left(1 - \frac{u}{v_0} + \frac{u^2}{v_0^2} - \frac{u^3}{v_0^3} + \frac{u^4}{v_0^4} - \dots \right).$$

Hence it follows

$$(14) \quad M(t) = T = t_0 \left(1 - \frac{M(u)}{v_0} + \frac{M(u^2)}{v_0^2} - \frac{M(u^3)}{v_0^3} + \frac{M(u^4)}{v_0^4} - \dots \right).$$

From this equation T may be computed when the characteristics of the motions are known. Before making this computation, we will simplify the above equation by expressing $M(u^s)$ by means of the characteristics.

Here u indicates the deviation from the *origin*. Designating the deviation from the *provisory mean* with ξ and this provisory mean with ξ_0 , we have the following relations:

$$\begin{cases} u = \xi + \xi_0; \\ u^2 = \xi^2 + 2\xi_0\xi + \xi_0^2; \\ u^3 = \xi^3 + 3\xi_0\xi^2 + 3\xi_0^2\xi + \xi_0^3; \\ u^4 = \xi^4 + 4\xi_0\xi^3 + 6\xi_0^2\xi^2 + 4\xi_0^3\xi + \xi_0^4. \end{cases}$$

As

$$\xi_0 = -0.5,$$

and according to the designation on page 24,

$$(15) \quad \begin{cases} M(\xi) = b; \\ M(\xi^2) = \nu_2'; \\ M(\xi^3) = \nu_3'; \\ M(\xi^4) = \nu_4', \end{cases}$$

we obtain the following expressions for the desired mean values:

$$(16) \quad \begin{cases} M(u) = b - 0.5; \\ M(u^2) = \nu_2' - b + 0.25; \\ M(u^3) = \nu_3' - 1.5\nu_2' + 0.75b - 0.125; \\ M(u^4) = \nu_4' - 2\nu_3' + 1.5\nu_2' - 0.5b + 0.0625. \end{cases}$$

We prefer, however, to express the above mean values in the moments about the mean. Without any difficulty we obtain with the help of (4) page 24.

$$(17) \quad \begin{cases} M(u) = u_0; \\ M(u^2) = \nu_2 + u_0^2; \\ M(u^3) = \nu_3 + 3u_0\nu_2 + u_0^3; \\ M(u^4) = \nu_4 + 4u_0\nu_3 + 6u_0^2\nu_2 + u_0^4. \end{cases}$$

Thus all the quantities in the right membrum of (14) are computed.

To obtain results directly comparable with those of MAUNDER, SPÖRER and others, it is more convenient to compute in the first place a value of the angular velocity from which T may be obtained directly, instead of computing the rotation-period from the characteristics of the motions in longitude. This value of the angular velocity is evidently not identical with $v_0 + u_0$ but is dependent on the dispersion, as well as on the skewness and excess.

If we suppose

T = the sidereal rotation period of the sun in mean solar days,

V = the corresponding angular velocity, expressed in minutes of arc per hour,
we have evidently according to (10)

$$VT = v_0 t_0.$$

Introducing here the value of T from the equation (14) we obtain for V the expression

$$(18) \quad V = \frac{v_0}{1 - \frac{M(u)}{v_0} + \frac{M(u^2)}{v_0^2} - \frac{M(u^3)}{v_0^3} + \frac{M(u^4)}{v_0^4} - \dots}.$$

The last four terms in the denominator are at most of the magnitude of $\frac{1}{40}$, $\frac{1}{200}$, $\frac{1}{2000}$, $\frac{1}{7000}$, respectively. By developing this expression and omitting the terms of the fifth and higher orders, we obtain for V , in using the equations (16) the following expression

$$(19) \quad V = v_0 + u_0 - \frac{\sigma^2}{v_0} + \frac{v_3 + u_0 \sigma^2}{v_0^2} - \frac{v_4 + 2u_0 v_3 + u_0^2 \sigma^2 - \sigma^4}{v_0^3} + \dots$$

Having thus computed V , the rotation period T is obtained from

$$(20) \quad T = \frac{900}{V}.$$

The above expression of V deserves a comment. The first term v_0 is nothing but the angular velocity of the system of co-ordinates expressed in minutes of arc per hour, besides which u_0 indicates the mean value of the motions in longitude referred to this system. Putting

$$V = v_0 + u_0$$

is evidently the same as writing

$$\frac{1}{M(v_0 + u)}$$

instead of

$$M\left(\frac{1}{v_0 + u}\right).$$

Compare with this the expression (13). The remaining terms of (19) arise from the moments of the second, third and fourth order, or in other words, from the dispersion, skewness and excess.

For the numerical use of the equation (19) we observe the following. In designating the mean error of a quantity A by $\varepsilon(A)$, we get from (19)

$$\varepsilon(V) = \varepsilon(u_0) \left(1 + \frac{\sigma_u^2}{v_0^2}\right) + \text{small quantities},$$

thus, with sufficient accuracy,

$$\varepsilon(V) = \varepsilon(u_0).$$

The calculations now show that $\varepsilon(u_0) \geq 0.1$. This circumstance permits us to omit in numerical computations the terms of the third and fourth order in (19), which

only amount to a few units in the third decimal. In computing the angular velocity of the sun at different latitudes, we therefore employ the sufficiently accurate formula

$$V = v_0 + u_0 - \frac{\sigma_u^2}{v_0}.$$

I write σ_u as it is a question of the dispersion of u or $\Delta\lambda$. Now we have previously computed the dispersion of x or $\Delta\lambda \cos \beta$. Employing the formula (11), we then obtain at last as the expression for V at an arbitrary latitude β

$$(20) \quad V = 35'.461 + x_0 \sec \beta - 0.0282 \sigma_x^2 \sec^2 \beta.$$

This value of V , introduced into the equation (20), gives the following expression for the sidereal rotation-period of the sun

$$(21) \quad T = 25^d.380 - 0.7157 x_0 \sec \beta + 0.0202 (x_0^2 + \sigma_x^2) \sec^2 \beta.$$

In the introduction a paper by the MAUNDERS¹ was mentioned. In this paper, as far as I can see, the authors have been guilty of an inadvertency. From the mean of the motions in longitude they have computed the rotation period directly, without regard to the mean error or the dispersion of the motions. As I have shown in the preceding lines, the dispersion of the motions will occur in the expression for the rotation period. The authors not having given this dispersion, but merely the averages, I have not computed the error in the period caused by this omission.

¹ Cited on page 10.

TABLE XIV.

The angular Velocity at different Latitudes.

Latitude-zones	First Period		Second Period	
	n	V	n	V
N_7	20	34.608	11	—
N_6	176	35.338	48	34.929
N_5	562	35.701	373	35.507
N_4	1081	35.941	644	35.579
N_3	1285	36.292	1239	35.867
N_2	709	36.281	1139	36.154
N_1	314	36.590	267	36.255
S_1	391	36.572	264	36.131
S_2	1108	36.334	888	36.347
S_3	1367	36.067	1181	35.885
S_4	1234	35.828	956	35.790
S_5	621	35.511	445	35.496
S_6	274	35.149	50	35.501
S_7	77	34.540	15	—

We shall now from the longitude motions of the spots numerically compute the angular velocity of the sun at different latitudes. From the values of x_0 and σ_x in table XI, V has been computed by means of equation (20). The computed values of V are given in table XIV. These values of the angular velocity we shall try to satisfy with a relation of the form

$$(22) \quad V = V_0 + V_1 \sin \beta + V_2 \sin^2 \beta,$$

where V_0 , V_1 , and V_2 are constants which we shall determine by means of the method of least squares. The term $V_1 \sin \beta$ has been introduced because the same northern and southern latitude have not the same angular velocity. For determining V_0 , V_1 and V_2 we have the following equations of condition:

	1886—1897		1898—1909	
	V	p	V	p
$V_0 + 0.53730 V_1 + 0.28869 V_2 =$	34.808	1	—	—
$V_0 + 0.46175 V_1 + 0.21321 V_2 =$	35.338	2	34.929	1
$V_0 + 0.38268 V_1 + 0.14644 V_2 =$	35.701	10	35.507	6
$V_0 + 0.30071 V_1 + 0.09043 V_2 =$	35.941	20	35.579	10
$V_0 + 0.21644 V_1 + 0.04685 V_2 =$	36.292	26	35.867	20
$V_0 + 0.13053 V_1 + 0.01704 V_2 =$	36.281	10	36.154	22
$V_0 + 0.04362 V_1 + 0.00190 V_2 =$	36.590	5	36.255	5
$V_0 - 0.04362 V_1 + 0.00190 V_2 =$	36.572	6	36.131	6
$V_0 - 0.13053 V_1 + 0.01704 V_2 =$	36.384	22	36.347	17
$V_0 - 0.21644 V_1 + 0.04685 V_2 =$	36.067	26	35.885	20
$V_0 - 0.30071 V_1 + 0.09043 V_2 =$	35.828	22	35.790	17
$V_0 - 0.38268 V_1 + 0.14644 V_2 =$	35.511	10	35.496	8
$V_0 - 0.46175 V_1 + 0.21321 V_2 =$	35.149	5	35.501	1
$V_0 - 0.53730 V_1 + 0.28869 V_2 =$	34.540	2	—	—

The weights p have been somewhat rounded in their calculation from the mean errors of V . From these equations of condition, the following systems of normal equations have been obtained.

$$\begin{aligned} \text{First period: } & \begin{cases} + 167 V_0 - 4.1340 V_1 + 12.0878 V_2 = + 253.3590; \\ - 4.1340 V_0 + 12.0878 V_1 - 0.5315 V_2 = - 2.0496; \\ + 12.0878 V_0 - 0.5315 V_1 + 1.4637 V_2 = + 14.8684. \end{cases} \\ \text{Second period: } & \begin{cases} + 133 V_0 - 2.2712 V_1 + 7.4776 V_2 = + 189.2972; \\ - 2.2712 V_0 + 7.4776 V_1 - 0.2914 V_2 = - 3.7728; \\ + 7.4776 V_0 - 0.2914 V_1 + 0.7109 V_2 = + 9.1070. \end{cases} \end{aligned}$$

In solving these equations we obtain at last the following expressions for the angular velocity of the sun as a function of the latitude.

First Period (1886—1897):

$$(23) \quad V = 36'.443 + 0'.240 \sin \beta - 5'.804 \sin^2 \beta.$$

Second Period (1898—1909):

$$(24) \quad V = 36'.223 - 0'.191 \sin \beta - 5'.390 \sin^2 \beta.$$

In these equations V is expressed in minutes of arc per hour. Expressing the angular velocities in degrees per mean solar day, we obtain the following equations.

First period:

$$(25) \quad V = 14^\circ.577 + 0^\circ.096 \sin \beta - 2^\circ.322 \sin^2 \beta.$$

Second period:

$$(26) \quad V = 14^\circ.489 - 0^\circ.076 \sin \beta - 2^\circ.156 \sin^2 \beta.$$

These two curves are represented in fig. 3. In this figure, the velocities obtained from the observations are in the first period indicated by small circles and in the

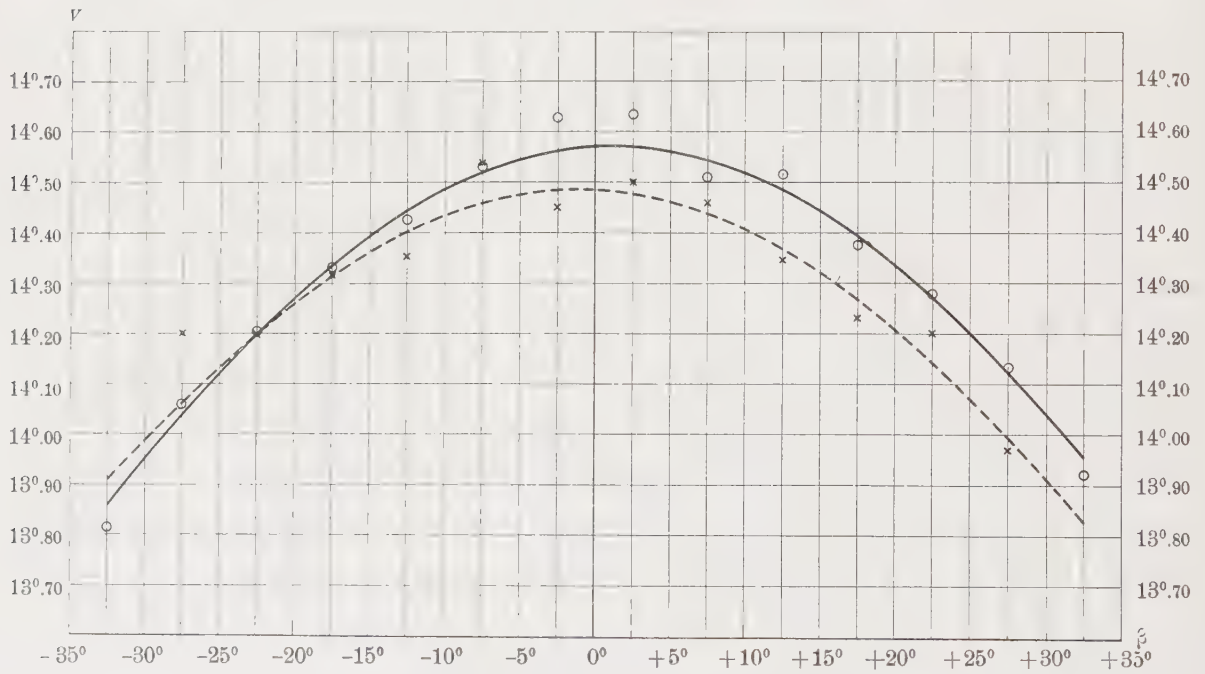


Fig. 3. The angular Velocity of the Sun at different Latitudes.

———— First Period (1886—1907). - - - - Second Period (1898—1909).

second by crosses. As shown in the figure the curves (25) and (26) express the variation of the angular velocity with the latitude in a satisfactory way.

During both these periods the zone of greatest angular velocity does not coincide with the equator. By derivating the expressions for V the latitude of the greatest angular velocity is obtained. We get

for the first period: $\beta = +1^{\circ}11'$,
 » » second » : $\beta = -1^{\circ}1'$.

The determination of these values is, however, not sufficiently accurate for us to conclude that the zone of the greatest angular velocity might be removable on the surface of the sun. If, however, we compute the values of V from (25) and (26) and form the differences between the first and second period, we obtain the values given in table XV. As is seen from this table, the angular velocity of the northern hemisphere seems to have decreased from the first to the second period, while the angular velocity of the southern hemisphere has remained almost unchanged. From fig. 3 or table XIV it follows that we arrive at the same result in using the observed angular velocities. That this circumstance can hardly be accidental appears from the fact that the differences between the angular velocities of the northern hemispheres during the two periods are generally greater than the mean errors of V , as is seen from table XI.

In connection with this it may be interesting to mention that the MAUNDERS in the paper cited above have found that »there seems to be an apparent slight shortening of the period for the northern hemisphere, whilst the southern is unchanged».

Now the investigation of the MAUNDERS comprises the years 1879—1901. The MAUNDERS' second period thus coincides with what I call the first period. Thus the two investigations comprise about three periods. Designating the angular velocity of a certain zone of the northern hemisphere during these three periods by V_1 , V_2 and V_3 respectively, we can comprehend these results in the inequality

$$V_1 < V_2 > V_3.$$

This inequality seems to indicate a periodical variation in the angular velocity of the northern hemisphere. For the southern hemisphere, however, no similar inequality seems to exist.

Should such a periodical variation in the angular velocity of the sun actually exist, which, of course, cannot hitherto be considered proved, WILCZYNSKI'S ¹ explanation of the periodicity of the spot-activity would possibly get a firmer basis.

TABLE XV.
 Computed angular Velocities at
 different Latitudes.

Latitude-zones	First Period	Second Period	Diff.
N_7	13.959	13.826	+0.133
N_6	14.126	13.994	+0.132
N_5	14.274	14.144	+0.130
N_4	14.396	14.271	+0.125
N_3	14.489	14.372	+0.117
N_2	14.551	14.442	+0.109
N_1	14.577	14.482	+0.095
S_1	14.569	14.488	+0.081
S_2	14.525	14.462	+0.063
S_3	14.447	14.404	+0.043
S_4	14.333	14.317	+0.021
S_5	14.200	14.202	-0.002
S_6	14.038	14.064	-0.026
S_7	13.855	13.908	-0.053

¹ E. J. WILCZYNSKI: On the Causes of the Sun-spot Period. Astrophysical Journal. Vol. VII. 1898.

TABLE XVI.
Means and Dispersions of the Motions in Longitude and Latitude for different Spot-sizes.

Latitude-zones	$0.800 \leq \omega < 1.139$						$1.200 < \omega < 1.599$						$1.600 < \omega < 1.999$					
	n	x_0	y_0	σ_x	σ_y	$\sigma_y : \sigma_x$	n	x_0	y_0	σ_x	σ_y	$\sigma_y : \sigma_x$	n	x_0	y_0	σ_x	σ_y	$\sigma_y : \sigma_x$
I N_7	3	+3.50	-2.25	2.45	2.04	0.83	7	-0.07	+0.54	3.81	0.84	0.22	1	-1.50	+0.75	0.00	0.00	—
N_6	16	-1.50	+0.09	2.67	1.97	0.74	26	+0.42	+0.13	3.43	1.28	0.97	49	+0.62	+0.16	2.68	1.52	0.57
N_5	46	+0.26	+0.09	1.97	1.34	0.68	115	+0.60	+0.05	3.11	1.90	0.61	96	+0.29	+0.01	2.47	1.38	0.56
N_4	108	+0.07	+0.23	2.81	1.52	0.54	155	+0.58	+0.05	3.17	1.49	0.47	236	+0.46	+0.08	2.48	1.15	0.46
N_3	111	+0.95	+0.11	3.05	1.45	0.48	174	+1.03	+0.07	3.07	1.38	0.45	244	+1.20	+0.06	2.08	1.09	0.52
N_2	56	+0.45	+0.03	2.99	1.64	0.55	135	+0.99	+0.04	3.58	1.29	0.36	171	+1.14	+0.18	2.71	1.24	0.46
N_1	31	+0.24	-0.04	2.78	1.58	0.56	48	+1.53	+0.16	2.98	1.52	0.51	86	+1.78	+0.12	2.85	1.26	0.44
I S_1	35	+1.56	+0.14	3.00	1.12	0.37	59	+1.08	+0.06	2.71	1.10	0.41	90	+1.45	-0.02	2.66	1.08	0.41
S_2	85	+1.24	+0.23	3.16	1.24	0.39	177	+1.27	0.00	2.93	1.48	0.51	234	+0.89	-0.05	2.13	1.17	0.55
S_3	133	+0.28	-0.07	2.86	1.89	0.47	234	+0.91	-0.05	2.91	1.35	0.46	259	+0.98	+0.05	2.49	1.14	0.46
S_4	140	+0.88	+0.11	2.42	1.49	0.62	225	+0.41	+0.09	2.92	1.45	0.50	218	+1.61	-0.08	2.25	1.21	0.54
S_5	62	-0.23	-0.21	3.51	1.52	0.43	125	+0.59	+0.09	2.98	1.48	0.50	120	+0.47	+0.02	2.96	1.32	0.45
S_6	37	-1.07	-0.02	2.98	1.70	0.57	56	+0.11	+0.28	2.74	1.31	0.48	44	+0.39	+0.06	2.44	1.39	0.57
S_7	7	+0.22	-0.04	2.12	1.41	0.67	19	-1.24	-0.09	2.00	1.17	0.58	17	-0.79	+0.46	2.67	1.45	0.54
II N_7	—	—	—	—	—	—	1	+2.50	+0.75	—	—	—	5	+1.90	+0.35	3.26	0.80	0.25
N_6	5	+1.50	-0.15	1.80	1.28	0.67	10	-1.40	+0.35	3.11	1.28	0.41	13	-0.65	-0.48	1.10	1.20	1.09
N_5	27	+0.72	-0.66	3.07	1.67	0.54	55	+0.12	-0.25	3.44	2.10	0.61	74	-0.03	+0.05	2.80	1.33	0.47
N_4	62	+0.06	+0.07	3.40	1.67	0.49	115	+0.02	+0.11	2.80	1.62	0.58	155	+0.63	+0.26	2.98	1.84	0.45
N_3	122	+0.21	+0.18	3.52	1.38	0.39	219	+1.15	+0.08	3.22	1.55	0.48	232	+0.41	-0.01	2.30	1.26	0.55
N_2	99	+1.27	-0.05	2.87	1.70	0.59	175	+0.60	+0.09	2.70	1.56	0.58	247	+0.83	+0.07	2.52	1.34	0.53
N_1	28	+0.54	-0.11	2.04	1.38	0.37	53	+1.05	+0.22	2.62	1.34	0.51	53	+0.97	+0.34	2.48	1.24	0.50
II S_1	20	+1.15	-0.02	2.26	1.62	0.72	56	+0.79	+0.18	2.71	1.49	0.55	70	+0.96	-0.07	2.22	1.24	0.56
S_2	79	+1.45	-0.17	2.98	1.87	0.63	164	+1.04	+0.19	2.99	1.52	0.51	216	+0.93	+0.22	2.40	1.20	0.50
S_3	114	+0.46	-0.13	3.28	1.52	0.46	182	+0.95	+0.18	2.75	1.46	0.53	261	+0.50	-0.09	2.75	1.21	0.44
S_4	71	+0.51	-0.40	3.25	1.71	0.53	175	+0.59	-0.11	2.88	1.54	0.53	233	+0.98	-0.01	2.82	1.40	0.50
S_5	48	+0.50	-0.01	2.98	1.42	0.48	82	+0.20	+0.04	2.61	1.25	0.48	115	+0.21	+0.06	2.43	1.42	0.58
S_6	5	+1.70	+0.65	3.92	0.97	0.25	14	+0.86	+0.75	3.18	1.54	0.48	9	-0.72	-0.08	2.53	1.99	0.79
S_7	5	-1.10	+0.95	3.20	0.87	0.27	7	-1.21	-0.61	3.24	0.85	0.29	2	+1.50	+1.00	3.00	0.75	0.25

Latitude-zones	$2.000 \leq \omega \leq 2.399$						$2.400 < \omega \leq 2.799$						$2.800 \leq \omega \leq 3.199$					
	n	x_0	y_0	σ_x	σ_y	$\sigma_y : \sigma_x$	n	x_0	y_0	σ_x	σ_y	$\sigma_y : \sigma_x$	n	x_0	y_0	σ_x	σ_y	$\sigma_y : \sigma_x$
I N_7	6	-1.50	+0.17	0.00	0.89	—	3	-1.50	-0.08	0.82	0.47	0.57	—	—	—	—	—	—
N_6	43	+0.15	+0.02	2.14	0.86	0.40	23	+0.80	-0.38	1.46	0.85	0.58	12	-0.17	-0.17	1.57	0.70	0.45
N_5	120	+0.49	+0.06	2.71	1.49	0.55	121	+0.24	-0.06	1.83	0.99	0.54	47	+0.73	+0.08	1.70	0.70	0.41
N_4	299	+0.78	-0.03	2.01	1.00	0.50	219	+0.84	-0.54	1.96	0.97	0.49	43	+1.43	-0.05	2.28	0.84	0.97
N_3	367	+0.85	+0.02	1.97	0.97	0.49	298	+0.69	+0.07	1.51	0.90	0.60	52	+1.48	+0.12	1.46	0.89	0.61
N_2	163	+1.13	+0.04	2.02	1.07	0.53	131	+0.96	-0.02	1.74	0.84	0.48	36	+1.14	-0.03	1.99	0.99	0.50
N_1	110	+1.23	-0.05	1.57	0.90	0.57	24	+1.50	+0.08	1.87	1.11	0.59	—	—	—	—	—	—
I S_1	120	+1.08	-0.06	2.06	0.81	0.39	66	+1.73	+0.20	1.77	0.86	0.49	6	+0.33	-1.17	1.07	0.73	0.68
S_2	315	+0.82	-0.06	1.66	0.90	0.54	190	+1.21	-0.06	1.56	0.92	0.59	64	+0.74	+0.04	1.30	0.84	0.65
S_3	371	+0.68	-0.01	1.81	1.05	0.58	214	+0.76	+0.01	1.78	0.89	0.50	103	+0.59	+0.03	1.72	0.81	0.47
S_4	310	+0.41	+0.01	2.10	0.91	0.43	247	+0.88	+0.05	1.66	0.81	0.49	52	+0.23	+0.10	2.14	0.86	0.40
S_5	139	-0.02	-0.02	1.95	1.06	0.54	121	+0.21	+0.07	1.56	0.88	0.56	37	+0.36	-0.24	1.70	1.29	0.76
S_6	60	+0.05	+0.12	1.88	0.96	0.51	41	-0.11	-0.06	1.56	1.28	0.82	17	+0.68	+0.10	2.04	1.20	0.59
S_7	6	+0.33	-0.42	1.70	0.90	0.53	8	-0.50	0.00	1.73	0.66	0.38	17	-1.44	+0.25	0.87	0.86	0.99
II N_7	5	-0.10	-0.05	1.74	0.51	0.29	—	—	—	—	—	—	—	—	—	—	—	—
N_6	14	-0.71	0.00	1.21	0.45	0.37	2	+1.50	-0.25	1.09	0.00	—	3	+3.17	-0.42	3.77	0.94	0.25
N_5	112	+0.30	+0.06	1.61	0.85	0.53	78	+0.37	0.30	1.99	1.05	0.53	16	+0.06	+0.06	1.70	0.95	0.50
N_4	186	+0.25	+0.13	1.96	1.18	0.60	83	+0.45	-0.13	2.10	0.96	0.46	22	-0.09	0.00	1.50	1.10	0.73
N_3	290	+0.52	+0.13	1.88	1.17	0.62	254	+0.56	+0.04	1.85	1.14	0.62	65	+0.50	+0.28	1.12	1.14	1.02
N_2	314	+0.72	+0.07	1.99	1.08	0.54	197	+1.05	-0.02	1.74	0.99	0.57	79	+0.84	+0.04	1.55	1.16	0.75
N_1	76	+0.96	+0.28	1.63	0.83	0.51	45	+1.12	+0.04	1.89	0.74	0.39	—	—	—	—	—	—
II S_1	68	+0.52	+0.25	1.42	0.93	0.65	31	+0.73	-0.49	1.24	0.82	0.66	12	+1.33	+0.29	1.57	0.63	0.40
S_2	234	+0.94	+0.19	1.64	0.88	0.53	132	+0.98	+0.03	1.62	0.91	0.56	27	+1.50	+0.14	1.41	0.99	0.70
S_3	313	+0.49	+0.11	1.89	1.14	0.60	177	+0.57	+0.10	1.75	1.09	0.62	78	+0.58	+0.03	1.95	0.80	0.41
S_4	244	+0.71	+0.11	2.11	1.02	0.48	133	+0.53	+0.15	1.85	0.97	0.72	61	+0.27	+0.31	1.56	0.91	0.58
S_5	121	0.00	-0.11	2.08	1.18	0.57	48	+0.88	+0.17	2.12	1.15	0.54	14	+0.07	-0.02	2.03	0.78	0.38
S_6	16	+0.06	+0.06	1.87	1.46	0.78	3	+0.50	-0.53	0.82	0.23	0.28	2	-0.50	+0.50	0.00	0.25	—
S_7	1	+3.50	+2.25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

D. The Motions in Longitude of Spots of different Sizes.

As previously stated, the dispersion of the motions decreases with increasing spot-area. Thus, if it should prove that the mean of the motions in longitude remains constant for spots of different sizes, it follows from (20) that the angular velocity will be greater, if determined from larger spots.

This question we shall now proceed to investigate. As a basis for this inquiry we use the values in table XVI. In this table the means and dispersions of the

TABLE XVII.
The Values of x_0 for different Spot-sizes. 1886—1909.

$\omega =$	1.00	1.40	1.80	2.20	2.60	3.00
N_1 and S_1	+0.879	+1.097	+1.315	+0.998	+1.345	+0.997
$N_2 \gg S_2$	+1.163	+0.974	+0.992	+0.866	+1.064	+0.948
$N_3 \gg S_3$	+0.455	+1.010	+0.775	+0.648	+0.643	+0.723
$N_4 \gg S_4$	+0.448	+0.429	+0.767	+0.558	+0.565	+0.494
$N_5 \gg S_5$	+0.225	+0.440	+0.262	+0.182	+0.341	+0.495
$N_6 \gg S_6$	-0.755	+0.143	+0.284	+0.004	+0.267	+0.530

TABLE XVIII.
The Values of σ_x for different Spot-sizes. 1886—1909.

$\omega =$	1.00	1.40	1.80	2.20	2.60	3.00
N_1 and S_1	2.604	2.750	2.591	1.730	1.731	1.421
$N_2 \gg S_2$	2.998	3.032	2.431	1.823	1.665	1.549
$N_3 \gg S_3$	3.207	2.997	2.426	1.888	1.713	1.632
$N_4 \gg S_4$	2.871	2.950	2.621	2.052	1.768	1.925
$N_5 \gg S_5$	2.978	3.017	2.674	2.126	1.830	1.744
$N_6 \gg S_6$	2.921	3.017	2.444	1.910	1.487	2.050

each such group have subsequently been divided into latitude-zones, after which the means and dispersions of the motions within each zone have been computed.

As the angular velocity V is determined from the formula

$$V = 35'.461 + x_0 \sec \beta - 0.0282 \sigma_x^2 \sec^2 \beta,$$

we have in the first place to examine whether the quantity

$$x_0 \sec \beta - 0.0282 \sigma_x^2 \sec^2 \beta$$

varies with ω within each zone.

motions in longitude as well as in latitude for different spot-areas are given. Although the motions in latitude are also introduced in the table, we shall not discuss them in this connection.

In making this table, the material of observation has been divided according to $\log \Delta (= \omega)$ in classes with a class-interval of 0.400. The spots for which ω is < 0.800 or > 3.200 being few, these groups have not been noted in the table. The observations within

Before proceeding with this investigation I brought together the corresponding zones in the northern and southern hemispheres during both periods. Thus, for each ω , I have combined N_1 and S_1 from the first period with N_1 and S_1 from the second and so forth. Let us for a moment indicate the values of x_0 in these four zones by x_1 , x_2 , x_3 and x_4 and the number of observations by n_1 , n_2 , n_3 and n_4 respectively. Then the values of x_0 and σ_x for all these four zones combined are computed from the easily deduced formulæ

$$(25) \quad \begin{cases} nx_0 = n_1x_1 + n_2x_2 + n_3x_3 + n_4x_4; \\ n\sigma_x^2 = n_1\sigma_1^2 + n_2\sigma_2^2 + n_3\sigma_3^2 + n_4\sigma_4^2, \end{cases}$$

where

$$n = n_1 + n_2 + n_3 + n_4.$$

In the expression for σ_x the terms containing the differences between the means are omitted.

TABLE XIX.

The Values of $V - 35'.461$ ($= \Delta V$) for different Spot-sizes. 1886—1909.

$\omega =$	1.00		1.40		1.80		2.20		2.60		3.00	
	n	ΔV	n	ΔV	n	ΔV	n	ΔV	n	ΔV	n	ΔV
N_1 and S_1	114	+0.688	216	+0.884	299	+1.156	374	+0.904	166	+1.262	18	+0.941
$N_2 \gg S_2$	319	+0.915	649	+0.718	868	+0.770	1026	+0.778	650	+0.993	206	+0.887
$N_3 \gg S_3$	480	+0.162	809	+0.769	996	+0.620	1341	+0.559	943	+0.577	298	+0.662
$N_4 \gg S_4$	381	+0.214	670	+0.180	842	+0.591	1039	+0.454	682	+0.495	178	+0.403
$N_5 \gg S_5$	183	-0.049	377	+0.176	405	+0.043	492	+0.048	368	+0.253	114	+0.371
$N_6 \gg S_6$	63	-1.156	106	-0.165	115	+0.106	133	-0.123	69	+0.222	34	+0.447

The computed values of x_0 and σ_x I have given in table XVII and XVIII. From table XVIII the previously mentioned fact of the dispersion decreasing with increasing spot-area, is clearly proved. From the values of x_0 and σ_x , given in these tables, I have computed the quantity

$$V - 35'.461 = x_0 \sec \beta - 0.0282\sigma_x^2 \sec^2 \beta,$$

in which V as before means the angular velocity at the latitude β . The results of these calculations are given in table XIX, where the quantity $x_0 \sec \beta - 0.0282\sigma_x^2 \sec^2 \beta$ is designated by ΔV . Within each zone the number of observations are also given. If we graphically represent the values of ΔV we see pretty clearly that, on an average, ΔV is increasing with ω . In order to get some idea of the size of this increase, I have within each zone expressed ΔV as a function of ω by means of the linear relation

$$\Delta V = a + b\omega.$$

The quantities a and b are computed with the help of the method of least squares. The weights are supposed to be about proportional to the number of observations. Thus, in the zone $N_8 + S_8$, I have assumed the weights to be 5, 8, 10, 13, 9, and 3 respectively. For the different zones I have in this way obtained the following expressions for the angular velocity, considered as a function of the spot-area.

$$\begin{array}{ll}
 \text{Zone } N_1 + S_1: & V = 35'.461 + 0'.717 + 0'.141\omega; \\
 \text{» } N_2 + S_2: & V = 35'.461 + 0'.646 + 0'.089\omega; \\
 \text{» } N_3 + S_3: & V = 35'.461 + 0'.417 + 0'.078\omega; \\
 \text{» } N_4 + S_4: & V = 35'.461 + 0'.095 + 0'.162\omega; \\
 \text{» } N_5 + S_5: & V = 35'.461 - 0'.124 + 0'.125\omega; \\
 \text{» } N_6 + S_6: & V = 35'.461 - 1'.192 + 0'.540\omega.
 \end{array}$$

The quantity b has proved positive in all the zones, and as for the size, it is about the same. The zone $N_6 + S_6$ forms, however, an exception, due to the fact of x_0 having got too small a value in this zone for $\omega = 1$, which must evidently be accidental.

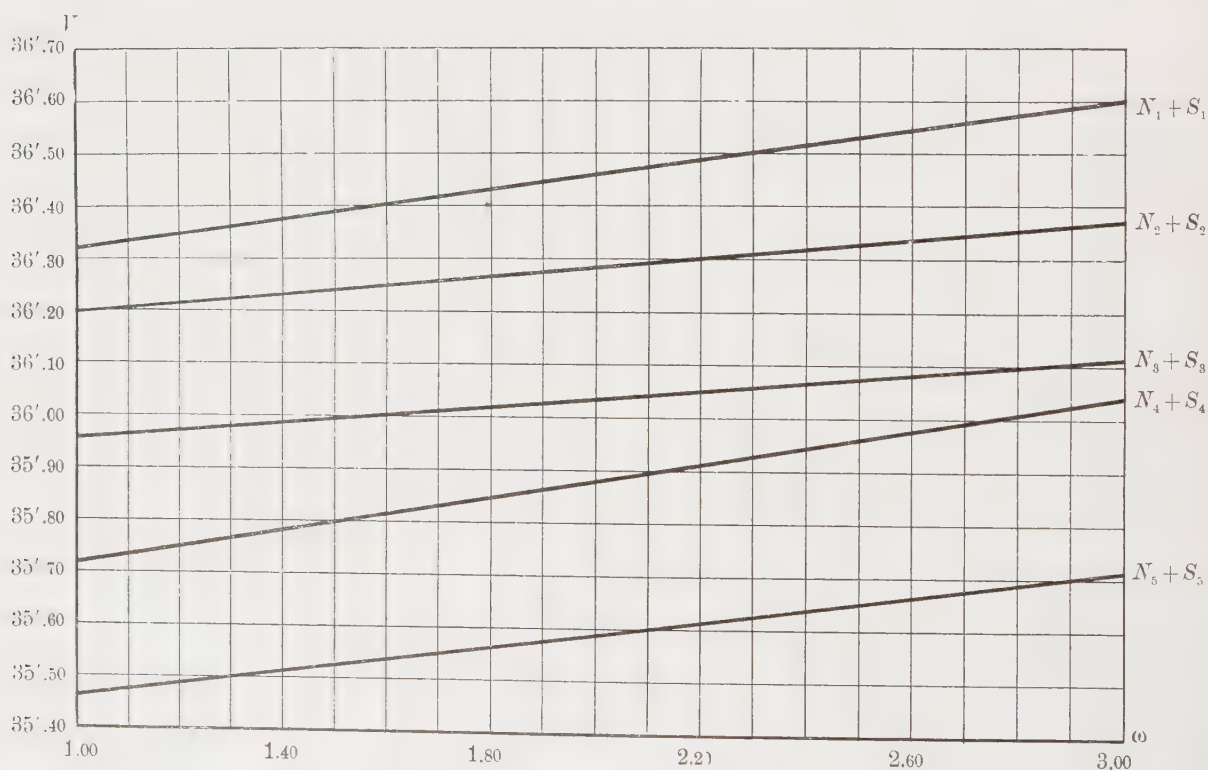


Fig. 4. The angular Velocity V at different Latitudes as a Function of ω .

In fig. 4 V is graphically represented as a linear function of ω . These lines are, undoubtedly, severally determined with considerably uncertainty, but as the

ordinate increases with increasing abscissa for all the lines, I consider it reasonable to draw the conclusion that a greater angular velocity, corresponding to a shorter rotation-period, is obtained from greater than from smaller spots, a result which in some degree may seem to be opposed to that of the MAUNDERS: »Spots of short duration tend to give a shorter rotation period than spots of long».

To conclude from the expressions for V that within each zone there should exist a linear relation between the angular velocity and $\log \Delta$ is naturally unjustified. These formulæ must only be regarded as rough approximations.

CHAPTER III.

The Latitude Motions of the Sun-spots.

A. Characteristics of the Motions in Latitude.

In treating the latitude motions of the spots exactly the same method has been employed as in treating the motions in longitude. However, in dividing the material of observation a class-interval of 0.5 instead of 1.0 has been adopted. In table XX I have given the distribution of these motions in latitude and in table XXI the computed values of the characteristics.

Mean and Dispersion. The problem of the latitude motions of the sun-spots has been the subject of several investigations. Whether an actual latitude drift does exist seems, however, from these investigations to be undecided. CARRINGTON, from the material embraced in his investigation (Nov. 9, 1853—Mars 24, 1861), has thought himself able to draw the conclusion that for spots north and south of $+20^\circ$ and -20° respectively the motions towards the Poles are predominant, while, on the contrary, the motions towards the equator predominate between 10° and 20° . The motions towards the Poles, according to CARRINGTON amount on an average to $2'$ a day and are about double the size of the opposite motions. From what I have been able to find in literature this assumption of CARRINGTON has not subsequently been confirmed. From the vast material of observation which is the basis of this investigation, nothing can be inferred that can support this assumption. Looking at table XXI where y_0 is the mean of the motions in latitude ($+$ for the northern and $-$ for the southern motions) no similar connection between the latitude motions and the latitudes of the spots appears. On the contrary, northern motions of the spots seem to be predominant in the northern as well as in the southern hemisphere.

Since this latitude drift assumed by CARRINGTON is also stated in text-books¹ as a fact which may be considered settled, I must once more emphasize that in the material embraced in this investigation, there is nothing whatever to support such

¹ See for example PRINGSHEIM: Physik der Sonne, page 59.

TABLE XX.

The Motions in Latitude of the Spots at different Latitudes.

Northern Hemisphere. First Period. 1886—1897.

Class-interval in $\Delta\beta = 0.5$. Provisory Mean = -0.25 .

Latitude	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
$0.0 < \beta < +2.4$	—	—	—	1	1	3	3	7	13	18	23	19	6	8	—	4	—	—	—	1	107
$+2.5 < \beta < +4.9$	—	—	1	2	2	6	9	16	20	43	56	20	13	7	6	2	1	3	—	—	207
$+5.0 < \beta < +7.4$	1	—	1	3	1	3	15	22	30	39	59	33	14	12	7	4	1	1	1	—	247
$+7.5 < \beta < +9.9$	—	1	1	5	6	10	19	24	66	69	111	74	30	24	11	9	2	—	—	—	462
$+10.0 < \beta < +12.4$	—	1	2	3	5	13	18	41	90	143	145	102	47	26	11	2	4	7	1	1	662
$+12.5 < \beta < +14.9$	1	1	3	3	3	8	22	41	86	114	151	97	46	22	10	9	2	3	1	—	623
$+15.0 < \beta < +17.4$	—	—	1	3	4	11	25	48	72	84	117	89	40	24	8	3	4	4	2	—	539
$+17.5 < \beta < +19.9$	1	2	1	2	7	8	23	39	83	93	122	72	40	26	9	3	4	2	3	2	542
$+20.0 < \beta < +22.4$	1	3	1	2	5	9	17	26	47	50	67	45	22	19	7	8	3	1	2	1	336
$+22.5 < \beta < +24.9$	2	2	2	2	3	7	15	10	30	33	39	32	23	9	9	2	—	3	2	1	226
$+25.0 < \beta < +27.4$	—	—	—	1	2	5	7	7	13	16	24	23	8	9	4	1	1	—	1	—	122
$+27.5 < \beta < +29.9$	—	—	1	—	2	1	—	3	7	7	17	8	4	4	—	—	—	—	—	—	54
$+30.0 < \beta < +32.4$	1	—	—	—	—	1	—	1	—	—	3	3	—	—	1	—	—	—	—	—	10
$+32.5 < \beta < +34.9$	—	—	—	—	—	—	—	—	—	3	4	2	1	—	—	—	—	—	—	—	10
Totals	7	10	14	27	41	85	173	285	560	709	938	619	294	190	83	47	22	24	13	6	4147

Southern Hemisphere. First Period. 1886—1897.

Class-interval in $\Delta\beta = 0.5$. Provisory Mean = -0.25 .

Latitude	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
$-0.1 > \beta > -2.5$	—	—	—	—	3	—	2	4	18	16	35	15	10	6	3	1	—	—	—	—	113
$-2.6 > \beta > -5.0$	—	—	—	—	4	8	8	18	45	58	72	28	21	10	4	2	—	—	—	—	278
$-5.1 > \beta > -7.5$	—	4	1	2	7	15	19	35	70	86	135	73	40	12	10	8	3	—	—	1	521
$-7.6 > \beta > -10.0$	1	2	1	2	5	15	23	34	77	102	156	87	50	16	7	6	2	—	1	—	587
$-10.1 > \beta > -12.5$	2	1	1	4	6	13	31	55	103	104	165	87	57	29	10	6	2	1	—	3	680
$-12.6 > \beta > -15.0$	1	2	1	5	3	13	31	53	93	114	167	98	59	22	7	4	2	6	4	2	687
$-15.1 > \beta > -17.5$	—	2	1	2	12	13	19	42	73	126	139	89	59	28	12	4	2	1	—	1	625
$-17.6 > \beta > -20.0$	2	2	—	3	8	12	22	53	76	106	135	99	49	20	9	4	2	5	—	2	609
$-20.1 > \beta > -22.5$	—	1	2	3	2	9	17	40	49	53	107	59	29	13	9	4	2	1	—	—	400
$-22.6 > \beta > -25.0$	3	—	3	2	5	4	8	13	38	29	43	32	19	8	5	4	2	—	1	2	221
$-25.1 > \beta > -27.5$	—	1	1	2	2	6	3	9	25	31	29	26	15	8	9	2	2	—	1	—	172
$-27.6 > \beta > -30.0$	—	—	—	1	3	4	5	10	14	12	17	12	13	3	5	2	1	—	—	—	102
$-30.1 > \beta > -32.5$	—	—	—	—	3	—	1	4	6	13	9	8	11	3	2	—	—	—	—	—	60
$-32.6 > \beta > -35.0$	—	—	—	—	—	1	—	5	—	2	3	4	—	1	—	—	1	—	—	—	17
$-35.1 > \beta > -37.5$	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	3
$-37.6 > \beta > -40.0$	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
$-40.1 > \beta > -42.5$	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	9	15	11	26	63	113	189	375	687	852	1213	718	432	179	93	48	21	14	7	11	5076

TABLE XX.
The Motions in Latitude of the Spots at different Latitudes.

Northern Hemisphere. Second period. 1898—1909.

Class-interval in $\Delta\beta = 0'.5$. Provisory Mean = $-0'.25$.

Latitude	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
$0.0 < \beta < +2.4$	—	—	—	—	—	—	—	2	4	9	16	9	3	2	—	1	—	—	—	—	46
$+2.5 < \beta < +4.9$	2	—	1	1	2	3	7	15	22	30	58	37	17	13	3	3	2	2	1	2	221
$+5.0 < \beta < +7.4$	1	2	7	3	3	7	14	32	59	77	85	70	35	15	10	7	4	2	1	2	436
$+7.5 < \beta < +9.9$	1	4	4	4	7	20	25	54	75	119	164	102	58	24	16	16	4	2	3	1	703
$+10.0 < \beta < +12.4$	1	1	2	4	8	15	20	57	68	105	135	87	59	42	18	6	5	4	7	2	646
$+12.5 < \beta < +14.9$	—	2	5	5	5	16	29	36	83	99	132	80	41	24	16	6	8	3	—	3	593
$+15.0 < \beta < +17.4$	—	—	1	4	3	10	15	18	35	64	79	46	24	10	9	6	6	2	3	—	335
$+17.5 < \beta < +19.9$	1	—	3	4	7	3	11	27	46	39	64	37	26	16	7	6	6	2	3	1	309
$+20.0 < \beta < +22.4$	—	4	1	2	2	9	9	14	40	35	56	27	22	13	6	3	2	—	1	1	247
$+22.5 < \beta < +24.9$	1	2	2	1	—	2	5	8	17	18	25	19	14	6	5	—	—	1	—	—	126
$+25.0 < \beta < +27.4$	—	—	—	—	—	1	1	4	3	12	9	4	1	2	1	—	—	—	—	—	38
$+27.5 < \beta < +29.9$	—	—	—	—	1	1	1	1	—	—	4	1	—	1	—	—	—	—	—	—	10
$+30.0 < \beta < +32.4$	—	—	—	—	—	—	—	—	2	3	2	2	2	—	—	—	—	—	—	—	11
Totals	7	15	26	28	38	87	137	268	454	610	829	521	302	168	91	54	37	18	19	12	3721

Southern Hemisphere. Second Period. 1898—1909.

Class-interval in $\Delta\beta = 0'.5$. Provisory Mean = $-0'.25$.

Latitude	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
$-0.1 > \beta > -2.5$	—	—	—	—	1	—	3	3	10	9	20	6	2	2	3	1	—	2	—	—	62
$-2.6 > \beta > -5.0$	—	2	2	—	3	4	5	9	22	40	54	29	13	8	7	—	4	—	—	—	202
$-5.1 > \beta > -7.5$	1	1	1	4	5	8	14	30	48	63	102	70	29	23	8	4	2	4	2	2	421
$-7.6 > \beta > -10.0$	—	2	—	3	5	7	13	35	63	74	113	78	33	18	10	7	2	2	1	1	467
$-10.1 > \beta > -12.5$	—	—	1	4	5	9	20	30	53	80	118	68	34	19	11	4	2	2	3	1	464
$-12.6 > \beta > -15.0$	1	3	4	4	10	27	37	43	83	111	148	102	77	34	14	7	7	2	1	2	717
$-15.1 > \beta > -17.5$	4	—	4	9	9	13	25	37	85	98	131	95	51	30	14	10	—	3	1	1	620
$-17.6 > \beta > -20.0$	2	1	3	—	5	9	16	24	35	59	60	48	38	16	11	6	—	2	1	—	336
$-20.1 > \beta > -22.5$	—	1	2	1	4	14	17	25	36	59	58	42	21	13	9	5	1	2	—	1	311
$-22.6 > \beta > -25.0$	—	—	3	1	—	2	3	9	18	20	27	17	17	7	3	3	2	1	—	1	134
$-25.1 > \beta > -27.5$	—	—	—	—	1	1	2	1	5	2	5	2	4	2	2	1	—	1	1	1	30
$-27.6 > \beta > -30.0$	—	—	—	—	1	—	—	3	2	5	4	3	1	1	—	—	—	—	—	—	20
$-30.1 > \beta > -32.5$	—	—	—	—	1	—	—	—	2	—	3	5	—	1	2	1	—	—	—	—	15
Totals	8	10	20	26	50	94	154	249	462	620	843	565	320	174	94	49	20	21	10	10	3799

a supposition. And I consider it the more essential that this supposition of CARRINGTON should not be adopted without further investigations, as it might easily lead theoretical researches on the conditions of the solar atmosphere to erroneous results.

TABLE XXI.
Characteristics of the Motions in Latitude.

Latitude-zones	First Period (1886—1897)					Second Period (1898—1909)				
	n	y_0	σ_y	S_y	E_y	n	y_0	σ_y	S_y	E_y
N_7	20	-0.075	+1.426	—	—	11	+0.205	+0.690	—	—
N_6	176	+0.076	+1.236	+0.124	+0.100	48	-0.136	+1.047	—	—
N_5	562	+0.012	+1.434	+0.052	+0.169	373	-0.012	+1.358	+0.230	+0.213
N_4	1081	+0.036	+1.185	-0.100	+0.205	644	+0.106	+1.364	-0.092	+0.149
N_3	1285	+0.062	+1.118	-0.013	+0.264	1239	+0.096	+1.306	-0.070	+0.180
N_2	709	+0.039	+1.194	+0.106	+0.136	1139	+0.050	+1.290	+0.078	+0.219
N_1	314	+0.052	+1.212	-0.101	+0.176	267	+0.218	+1.252	+0.012	+0.389
S_1	391	+0.004	+0.977	+0.068	+0.078	264	+0.097	+1.206	+0.112	+0.255
S_2	1108	-0.011	+1.122	+0.173	+0.230	888	+0.108	+1.212	-0.029	+0.249
S_3	1367	-0.008	+1.176	-0.054	+0.274	1181	+0.052	+1.252	+0.035	+0.170
S_4	1234	+0.024	+1.150	+0.069	+0.231	956	+0.026	+1.298	+0.200	+0.166
S_5	621	-0.018	+1.264	+0.118	+0.229	445	+0.030	+1.315	-0.025	+0.139
S_6	274	+0.070	+1.327	+0.074	+0.062	50	+0.320	+1.559	—	—
S_7	77	+0.114	+1.172	—	—	15	+0.616	+1.335	—	—

The dispersions of the latitude motions prove, on the whole, to be about half the size of the dispersions in longitude. This fact seems to indicate that the currents in the solar atmosphere parallel to the equator are greater than those in a direction perpendicular to the equator. Table XXII gives the quotient between σ_y and σ_x . As is evident from the table this quotient does not vary with the latitude. Hence we may infer that the relation between the sizes of the currents parallel and perpendicular to the equator is the same at all latitudes within the spot-zones.

As regards the *skewness*, it proves to be alternatively positive and negative. One might perhaps say that a positive skewness is predominant. If, however, we take into consideration the mean error of the skewness, which is very great —

TABLE XXII.
The Values of $\sigma_y : \sigma_x$ at different Latitudes.

Latitude-zones	First Period (1886—1897)		Second Period (1898—1909)	
	n	$\sigma_y : \sigma_x$	n	$\sigma_y : \sigma_x$
N_7	20	0.475	11	0.255
N_6	176	0.474	48	0.440
N_5	562	0.587	373	0.552
N_4	1081	0.485	644	0.518
N_3	1285	0.504	1239	0.534
N_2	709	0.448	1139	0.566
N_1	314	0.496	267	0.560
S_1	391	0.392	264	0.577
S_2	1108	0.512	888	0.522
S_3	1367	0.497	1181	0.503
S_4	1234	0.493	956	0.531
S_5	621	0.495	445	0.542
S_6	274	0.556	50	0.587
S_7	77	0.559	15	0.394

1.9325 : $\sqrt{n}$ — we cannot attach any importance to this slight positive skewness. Even as regards the latitude motions we are practically entitled to put $S = 0$.

The same pervading positive *excess* which we have already met with in the study of the longitude motions, we also find in the motions in latitude. As it appeared that the dispersions also of the latitude motions diminished with increasing spot-area, I have computed the characteristics of the motions of the spots of different sizes. From reasons before mentioned the material has been divided according

TABLE XXIII.

Characteristics of the Latitude Motions for different Spot-sizes.

First Period (1886—1897).

Logarithm of Spot-area	Northern Hemisphere					Southern Hemisphere				
	n	y_0	σ_y	S_y	E_y	n	y_0	σ_y	S_y	E_y
$0.800 < \omega < 1.199$	371	+0.096	+1.523	−0.019	+0.151	499	+0.032	+1.426	+0.132	+0.089
$1.200 < \omega < 1.599$	658	+0.070	+1.495	+0.013	+0.108	895	+0.043	+1.433	+0.003	+0.169
$1.600 < \omega < 1.999$	883	+0.097	+1.214	−0.018	+0.112	982	−0.006	+1.204	+0.142	+0.183
$2.000 < \omega < 2.399$	1108	+0.010	+1.053	+0.029	+0.250	1321	−0.019	+0.959	+0.093	+0.163
$2.400 < \omega < 2.799$	819	−0.005	+0.932	+0.073	+0.063	887	+0.021	+0.894	+0.105	+0.246
$2.800 < \omega < 3.199$	148	+0.028	+0.852	+0.245	+0.141	296	+0.004	+0.948	+0.214	+0.195

Second Period (1898—1908).

Logarithm of Spot-area	Northern Hemisphere					Southern Hemisphere				
	n	y_0	σ_y	S_y	E_y	n	y_0	σ_y	S_y	E_y
$0.800 < \omega < 1.199$	342	+0.035	+1.626	+0.090	+0.084	342	−0.133	+1.530	+0.041	+0.071
$1.200 < \omega < 1.599$	628	+0.076	+1.604	+0.025	+0.079	680	+0.096	+1.486	+0.029	+0.110
$1.600 < \omega < 1.999$	779	+0.092	+1.918	−0.054	+0.141	906	+0.028	+1.806	+0.041	+0.155
$2.000 < \omega < 2.399$	998	+0.114	+1.090	−0.090	+0.266	997	+0.102	+1.058	+0.013	+0.227
$2.400 < \omega < 2.799$	659	+0.001	+1.063	+0.038	+0.355	524	+0.090	+1.011	+0.175	+0.162
$2.800 < \omega < 3.199$	185	+0.115	+1.130	+0.226	+0.323	194	+0.168	+0.892	−0.044	+0.107

to the logarithm of the spot-area, in the same manner as in discussing the positive excess of the motions in longitude.

In table XXIII I have brought together the characteristics of the motions of the different spot-sizes. The spots for which ω is < 0.8 or > 3.2 are not given in the table, because of their being too few to admit of a calculation of the higher characteristics. As seen from the table the excess of the motions of the different groups is of about the same size as it was before the dividing of the material.

Hence it follows that the fact of the dispersion varying with the spot-area cannot be assumed to be the cause of the positive excess.

In an investigation into the position of the sun's axis¹, mentioned in the introduction, DYSON and MAUNDER have tabulated the distribution of the latitude motions of 867 recurrent spots from 1874—1911, which distribution has a certain bearing upon the question of the positive excess now under discussion. We borrow from this paper table XXIV containing the distribution of the latitude motions. Calculating the characteristics of this distribution we find, on an average, a northern motion even of these recurrent spots. The skewness as well as the excess are on the contrary = 0. The distribution is thus quite normal. — Here it will not be out of place to make a remark. In the paper mentioned the authors give a graphical representation of the distribution in question (Fig. 5). Here the scale of the ordinate as well as of the abscissa is arbitrarily chosen, so that from the figure we do not get any idea as to whether the distribution is normal or not. It would be still more advantageous if a system of normal co-ordinates were employed in similar representations.

As suggested by CHARLIER (see page 1) one might appropriately employ the dispersion as the unit of the abscissa. In order that the frequency-curve may not be too flat the function of distribution is multiplied by 5, whereby the maximum ordinate amounts to about 2 (= 1.9945).

If we make use of these normal co-ordinates, the distribution will have the appearance shown in fig. 6. Here the curve represents the normal distribution. As it appears from this figure the observed frequencies are very nearly in agreement with the normal distribution, which is also evident from the fact of the skewness as well as the excess being equal to zero.

¹ See page 12.

TABLE XXIV.
Proper Motion in Latitude of
Recurrent Spot Groups.
(1874—1911. DYSON and MAUNDER.)

Proper Motion in a Rotation			N:o of Observations
From	$+5.5$ to $+5.0$		1
	$+5.0$ to $+4.5$		1
	$+4.5$ to $+4.0$		2
	$+4.0$ to $+3.5$		3
	$+3.5$ to $+3.0$		7
	$+3.0$ to $+2.5$		27
	$+2.5$ to $+2.0$		48
	$+2.0$ to $+1.5$		51
	$+1.5$ to $+1.0$		90
	$+1.0$ to $+0.5$		110
	$+0.5$ to 0.0		113
	0.0 to -0.5		125
	-0.5 to -1.0		85
	-1.0 to -1.5		81
	-1.5 to -2.0		51
	-2.0 to -2.5		33
	-2.5 to -3.0		26
	-3.0 to -3.5		9
	-3.5 to -4.0		2
	-4.0 to -4.5		2

Mean = $+0.082$

Dispersion = $+1.469$

Skewness = $+0.003$

Excess = -0.009



Fig. 5. Proper Motion in Latitude of Recurrent Spot Groups.

(The Figure is taken from the Paper of DYSON and MAUNDER)

What is chiefly of interest to us in this connection is the fact that, while the excess is thoroughly positive in the motions we have investigated, no such positive excess exists in the material of DYSON and MAUNDER. The main difference between this material and that of the author consists in the latitude motions of the former being computed from an interval of one day. How is the difference between the distributions of the motions then to be explained? One might possibly think that the positive excess may have arisen in the following way. Suppose the currents in the solar atmosphere to have an arbitrary velocity, and further suppose that these velocities form a normal distribution. Thus, if the spots moved with the same velocity as these currents, the distribution of the motions of the spots would also be normal. However, it now seems very plausible that the velocities of the currents are not immediately reached. There should evidently arise a certain acceleration in

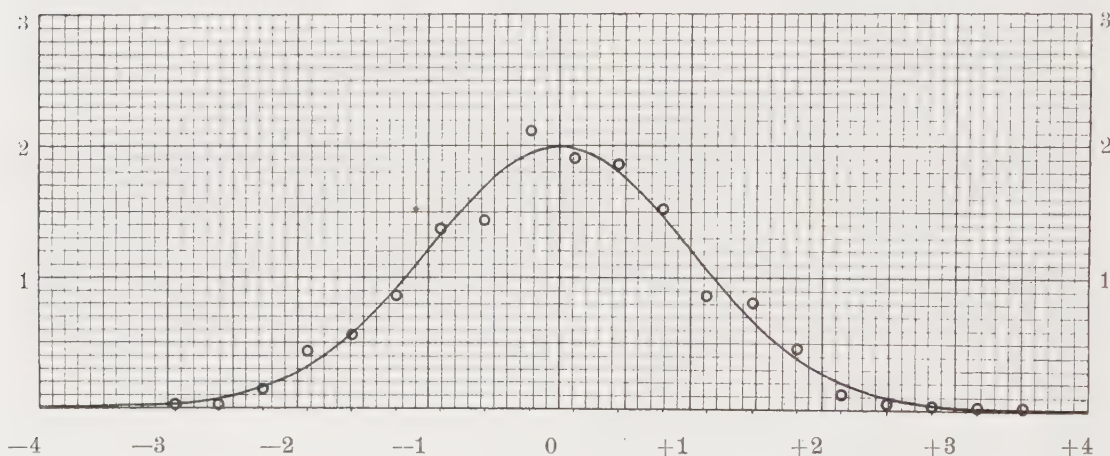


Fig. 6. Proper Motion in Latitude of Recurrent Spot Groups compared with a normal Distribution.

the motions of the spots till the velocities of the spots become approximately as large as that of the currents. Now, if the velocities are determined from shorter intervals, there is a possibility of a greater number of small velocities being obtained than those corresponding to the normal distribution.

Too great importance, however, should not be attached to this attempt to explain the positive excess. The actual cause of the excess must rather be regarded as still uncertain.

B. On the Position of the Sun's Axis.

In the introduction, page 8, I mentioned some of the more important determinations of the rotation axis of the sun. The heliographic co-ordinates of the spots published in the Greenwich Observations are calculated by employing CARRINGTON's values: $I = 7^{\circ}15'$; $L = 73^{\circ}40'$. In using the observations of 867 recurrent spots

from 1874—1911, published in the Greenwich Observations, DYSON and MAUNDER, as stated on page 11, have computed the following corrections of these values:

$$\begin{cases} \Delta I = -4'.4; \\ \Delta L \sin I = +1'.7. \end{cases}$$

Even if these corrections should in no wise be considered definite, the paper by DYSON and MAUNDER just mentioned is of some interest, since it clearly proves that the position of the rotation axis can only be determined with considerable inaccuracy.

It is easily seen that if the axis is erroneously determined, this will cause an annual period in the latitude motions of the spots. Now DYSON and MAUNDER have not found any such period in their material of observation. Hence it seems as if we should be in a position to infer that the inclination did not require any sensible correction. Their following treatment of the latitude motions gives, nevertheless, as the result the above values.

These values of ΔI and $\Delta L \sin I$ are averages of values computed for each year. About the sun-spot minima three years are usually taken together. In every such group the number of observations varies between 50 and 150 or thereabouts. In computing the final averages the years 1874—77 have been omitted, during which time the observations give the values $\Delta I = +12'.4$ and $\Delta L \sin I = -9'.8$, which considerably differ from the mean. The authors have not calculated any mean errors. If we compute

the mean errors of the results from the 23 groups, in which the material is divided, we obtain, *without considering the weights*,

$$\begin{cases} \Delta I = -4'.6 \pm 1'.2 \\ \Delta L \sin I = +1'.5 \pm 1'.3 \end{cases}$$

Hence it follows that one can hardly attach any importance at least to the correction of L . As regards the correction of I it varies in such a degree that it seems impossible from the motions of the spots to determine the position of the sun's axis more accurately than has hitherto been done. This appears most clearly

TABLE XXV.

Values of $\Delta L \sin I$ and ΔI according to Dyson and Maunder.

Period	Weight	$\Delta L \sin I$	ΔI
1874—1877	282	— 9'.8	+12'.4
1878—1880	208	— 1.1	— 4.3
1881	304	+ 2.4	— 9.2
1882	345	+ 9.1	— 9.7
1883	448	— 0.2	— 4.8
1884	460	+11.7	— 0.5
1885	379	+11.5	— 4.5
1886—1888	339	— 1.8	— 8.1
1889—1891	355	— 3.9	— 4.2
1892	541	+ 6.1	— 9.4
1893	694	+ 1.6	— 8.6
1894	644	+ 3.1	+ 0.4
1895	497	+ 5.7	— 9.7
1896	251	— 4.2	—11.8
1897	221	+ 8.2	—12.8
1898—1900	311	— 4.3	—13.5
1901—1903	203	+ 5.3	— 1.4
1904	288	+ 1.8	+ 5.6
1905	220	+ 8.7	— 6.5
1906	407	— 5.1	— 2.9
1907	473	+ 0.9	+ 8.3
1908	372	— 6.9	— 4.0
1909	369	— 9.7	+ 2.0
1910—1911	217	— 3.7	+ 3.2
1878—1888	2484	+ 5.3	— 5.7
1889—1900	3535	+ 2.1	— 7.7
1901—1911	2552	— 2.0	+ 1.8
1878—1911	8546	+ 1.7	— 4.4

TABLE XXVI.

The Motions in Latitude at different Times in the Year.*Northern Hemisphere. First Period. 1886—1897.*Class-interval in $\Delta\beta = 0'.5$. Provisory Mean = $-0'.25$.

Part of the Year	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
1	—	3	4	2	2	5	15	28	60	49	66	48	24	17	11	3	—	5	1	—	343
2	1	—	1	3	2	6	16	26	51	56	89	66	28	19	7	1	1	1	1	—	375
3	—	—	2	6	3	5	21	35	42	73	102	62	47	22	8	9	1	3	1	—	442
4	1	1	1	—	4	7	9	24	51	82	73	74	29	16	6	4	5	—	4	1	392
5	2	—	1	5	4	13	15	27	46	90	116	82	42	27	10	5	1	2	1	—	489
6	2	2	2	3	4	6	10	28	50	78	97	69	29	14	7	6	2	1	1	1	412
7	—	—	—	2	4	10	10	21	64	74	116	70	29	17	4	2	4	4	1	1	433
8	—	4	1	2	4	6	27	26	54	78	107	47	26	21	9	6	1	—	—	2	421
9	1	—	—	3	5	14	29	27	65	65	85	33	21	17	12	6	4	5	—	—	392
10	—	—	2	1	9	13	21	43	77	64	87	68	19	20	9	5	3	3	3	1	448
Totals	7	10	14	27	41	85	173	285	560	709	938	619	294	190	83	47	22	24	13	6	4147

*Southern Hemisphere. First Period. 1886—1897.*Class-interval in $\Delta\beta = 0'.5$. Provisory Mean = $-0'.25$.

Part of the Year	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
1	—	1	—	2	8	9	17	45	72	100	113	61	44	13	8	3	3	3	2	1	505
2	1	1	1	4	7	9	17	26	65	89	129	68	45	16	13	6	2	4	—	1	504
3	1	—	—	3	6	5	12	29	55	59	100	59	38	18	10	6	—	—	2	2	405
4	—	—	1	2	3	11	12	41	62	77	122	84	50	16	12	8	3	1	—	1	506
5	1	3	1	3	8	8	18	53	71	78	147	87	47	15	13	4	3	1	—	—	561
6	2	2	2	2	6	12	27	48	80	114	160	98	67	35	10	4	2	3	1	2	677
7	1	2	2	3	6	11	29	33	85	104	143	73	40	19	9	7	1	2	1	1	572
8	1	1	1	3	8	14	19	28	49	53	86	66	46	18	8	3	2	—	—	—	406
9	—	3	—	2	6	22	14	28	70	80	99	56	33	11	7	5	2	—	1	1	440
10	2	2	3	2	5	12	24	44	78	98	114	66	22	18	3	2	3	—	—	2	500
Totals	9	15	11	26	63	113	189	375	637	852	1213	718	432	179	93	48	21	14	7	11	5076

TABLE XXVI. Cont.

The Motions in Latitude at different Times in the Year.*Northern Hemisphere. Second Period. 1898—1909.*Class-interval in $\Delta\beta = 0'.5$. Provisory Mean = $-0'.25$.

Part of the Year	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
1	—	1	2	5	5	12	20	30	38	75	83	69	30	26	13	7	2	1	1	1	421
2	—	3	3	—	4	8	11	29	39	60	81	48	30	12	9	3	3	2	5	1	351
3	—	1	—	2	3	5	8	19	31	52	68	48	32	17	6	2	2	1	1	1	302
4	1	3	1	4	5	8	11	22	49	44	72	30	32	19	5	5	6	1	1	1	320
5	—	2	3	—	3	6	14	25	27	52	81	42	31	22	9	2	5	—	1	4	329
6	—	1	3	1	3	6	18	30	62	69	86	54	32	18	12	4	2	1	2	—	404
7	1	2	1	4	4	11	10	15	47	58	92	54	17	15	6	2	4	2	1	1	347
8	—	—	2	5	2	4	12	24	48	54	70	51	23	9	5	7	2	2	2	—	322
9	3	—	5	4	6	18	16	45	48	73	94	72	39	17	18	17	8	3	3	1	490
10	2	2	6	3	3	9	17	29	62	73	102	53	36	13	8	5	3	5	2	2	435
Totals	7	15	26	23	38	87	137	268	454	610	829	521	302	168	91	54	37	18	19	12	3721

*Southern Hemisphere. Second Period. 1898—1909.*Class-interval in $\Delta\beta = 0'.5$. Provisory Mean = $-0'.25$.

Part of the Year	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
1	1	1	—	2	3	7	13	18	36	55	76	51	42	14	5	1	1	4	2	1	333
2	—	1	2	2	8	9	17	42	60	78	98	56	30	21	12	4	5	3	1	2	451
3	1	1	3	3	4	7	20	21	49	73	105	71	38	20	8	—	2	1	2	2	431
4	1	1	3	5	5	4	13	24	39	55	77	57	25	15	10	10	2	1	1	1	349
5	—	1	1	4	6	10	14	14	36	52	61	52	20	13	8	9	—	1	2	1	305
6	1	—	4	3	5	12	14	18	48	54	66	59	34	14	8	2	2	3	—	—	347
7	1	1	—	1	2	4	14	24	42	64	91	65	40	16	7	5	1	—	—	1	379
8	1	—	2	1	6	13	14	33	46	82	111	57	30	22	9	4	—	—	—	—	431
9	—	2	4	2	5	12	11	25	54	47	78	50	33	20	14	9	2	3	2	1	374
10	2	2	1	3	6	16	24	30	52	60	80	47	28	19	13	5	5	5	—	1	399
Totals	8	10	20	26	50	94	154	219	462	620	843	565	320	174	94	49	20	21	10	10	3799

from table XXV, which I have borrowed from the paper of DYSON and MAUNDER. That any periodicity should exist in these variations does not seem probable. On the contrary it seems as if the inclination had somewhat increased during the last decennia. The sun being gaseous, several circumstances seem to indicate that one cannot speak of a rotation axis in the same sense as in a rigid body.

We will now proceed to investigate to what extent the material of observation dealt with in this paper indicates any error in the position of the rotation axis of the sun. To begin with we shall investigate whether any annual period does exist in the motions of the spots. As the time for the appearance of the spots in our card catalogue is stated in hundredths of a year, I have divided the material into ten groups for each year. In doing this I have for the sake of comparison, taken the two periods as well as the two hemispheres separately. In table XXVI the

TABLE XXVII.

Mean Values of the Motions in Latitude at different Times in the Year.

Part of the Year	First Period (1886—1897)				Second Period (1898—1909)				1886—1909			
	Northern Hemisphere		Southern Hemisphere		Northern Hemisphere		Southern Hemisphere		n	y_0	σ_y	$\varepsilon(y_0)$
	n	y_0	n	y_0	n	y_0	n	y_0				
1	343	—0.003	505	—0.012	421	+0.082	333	+0.163	1602	+0.051	+1.237	0.031
2	375	+0.052	504	+0.080	351	+0.096	451	+0.028	1681	+0.063	+1.225	0.030
3	442	+0.131	405	+0.116	302	+0.180	431	+0.082	1580	+0.122	+1.192	0.030
4	392	+0.144	506	+0.138	320	+0.033	349	+0.096	1567	+0.109	+1.242	0.031
5	489	+0.112	561	—0.002	329	+0.194	305	+0.065	1684	+0.082	+1.226	0.030
6	412	+0.046	677	+0.076	404	+0.042	347	+0.006	1840	+0.049	+1.202	0.028
7	433	+0.112	572	—0.036	347	+0.022	379	+0.154	1731	+0.054	+1.150	0.028
8	421	—0.025	406	—0.004	322	+0.045	431	—0.010	1580	—0.001	+1.186	0.030
9	392	—0.082	440	—0.102	490	+0.116	374	+0.126	1696	+0.016	+1.333	0.033
10	448	—0.056	500	—0.176	445	+0.005	399	—0.028	1782	—0.069	+1.292	0.031
	4147	+0.045	5076	+0.009	3721	+0.079	3799	+0.065	16743	+0.046	+1.231	0.008

result of this division is given. The computed means of the latitude motions are brought together in table XXVII.

During the period 1886—1897 an obvious periodicity appears, especially in the northern hemisphere. On the other hand this periodicity does not seem to exist, at least not so conspicuously, during the following period, which indicates that the axis of the sun does not appear to occupy a fixed position. Still better does this periodicity of the motions appear, when they are graphically represented. Although it seems as if we could not expect to obtain any better determination of the position of the sun's axis from the latitude motions, we will, however, on account of these motions inquire further into this matter.

We designate the values of the inclination and of the longitude of the ascending node, employed in the computation of the heliographic co-ordinates, by I and L

and the actual values by $I + \Delta I$ and $L + \Delta L$. The problem now is to determine the corrections ΔI and ΔL from the motions of the spots. Suppose the observed co-ordinates of a spot to be β_1, λ_1 one day, and β_2, λ_2 the following day. A change in the position of the rotation axis will then cause a change in the latitude of the spot, which is obtained from the easily deduced formula

$$\beta - \beta_1 = \Delta L \sin I \cos \lambda_1 - \Delta I \sin \lambda_1.$$

For the following day we obtain

$$\beta - \beta_2 = \Delta L \sin I \cos \lambda_2 - \Delta I \sin \lambda_2.$$

We thus leave the proper motion of the spot out of consideration and assume the actual latitude of the spot to remain unchanged from one day to the other. Subtracting the equations we get

$$\beta_2 - \beta_1 = -\Delta L \sin I (\cos \lambda_2 - \cos \lambda_1) + \Delta I (\sin \lambda_2 - \sin \lambda_1).$$

Now we introduce the notation

$$(27) \quad \begin{cases} p = \Delta L \sin I; \\ q = \Delta I, \end{cases}$$

and obtain after a short reduction

$$(28) \quad \beta_2 - \beta_1 = 2 \sin v (p \sin u + q \cos u),$$

where

$$(29) \quad \begin{cases} 2u = \lambda_2 + \lambda_1; \\ 2v = \lambda_2 - \lambda_1. \end{cases}$$

Since λ_1 and λ_2 designate the longitudes of the spot on two consecutive days, $2v$ is nothing but the daily rate of rotation of the sun at the latitude in question. We have now taken all the latitude-zones together and so we may approximately put $2v = 14^{\circ}.4$ or

$$v = 7^{\circ}.2.$$

Instead of employing u , which quantity represents the mean of the longitudes on two consecutive days, we refer the position of the spot to the central meridian and write

$$(30) \quad u = u_0 + l$$

where u_0 is the longitude of the centre of the sun's visible disk, and l the longitude of the spot referred to the central meridian.

Now we have only included in our investigation such spots as are situated within 60° from the central meridian. Since l is continually changing while the spot is moving across the sun's surface, $\beta_2 - \beta_1$ will also be changed. We will therefore compute a mean value of $\beta_2 - \beta_1$. This computation may with sufficient accuracy be made in the following manner.

From the equations (28) and (30) we obtain

$$\beta_2 - \beta_1 = 2 \sin v [(p \sin u_0 + q \cos u_0) \cos l + (p \cos u_0 - q \sin u_0) \sin l].$$

Seeing that the spots are approximately situated symmetrically on both sides of the central meridian, $\sin l$ will, on an average, be equal to zero. Thus we have only

$$\beta_2 - \beta_1 = 2 \sin v (p \sin u_0 + q \cos u_0) \cos l.$$

Here l varies, on an average, from $-(60^\circ - 7^\circ.2)$ to $+(60^\circ - 7^\circ.2)$. For a certain u_0 we thus obtain, designating the mean value of $\beta_2 - \beta_1$ by $24y_0$, y_0 being the latitude motion in minutes of arc per hour,

$$(31) \quad p \sin u_0 + q \cos u_0 = ky_0$$

where

$$k = \frac{24}{2 \sin v \cdot M(\cos l)} = 110.9.$$

From equation (31) p and q are to be computed. For the sake of comparison we determine p and q from the motions in the two hemispheres during both periods

TABLE XXVIII.

Values of u_0 , $M(\sin u_0)$ and $M(\cos u_0)$ at different Times in the Year.

Time in the Year	u_0	$M(\sin u_0)$	$M(\cos u_0)$
0.00	27.0		
0.10	64.1	+ 0.7017	+ 0.6884
0.20	100.8	+ 0.9747	+ 0.1291
0.30	136.8	+ 0.8621	-- 0.4740
0.40	172.1	+ 0.4246	-- 0.8883
0.50	207.0	-- 0.1634	-- 0.9711
0.60	241.9	-- 0.6897	-- 0.7030
0.70	277.1	-- 0.9081	-- 0.1794
0.80	313.1	-- 0.8910	+ 0.4172
0.90	349.8	-- 0.4699	+ 0.8637
1.00	27.0	+ 0.1433	+ 0.9723

separately. For determining these quantities we thus obtain four systems of equations with ten equations in each system. As regards the values of u_0 we observe that u_0 is nothing but the heliographic longitude of the earth reckoned from the ascending node of the sun's equator. Seeing that the inclination between the sun's equator and the ecliptic is only $7^\circ 15'$, we may in computing u_0 at different times of the year employ directly the longitude of the earth, the correction being at maximum only $tg \frac{2I}{2}$ or $0^\circ.23$.

Designating the longitude of the sun by $\odot$ we immediately obtain

$$u_0 = \odot - 180 - L.$$

For L I have employed CARRINGTON's value $L = 73^\circ 40' 1850.0$. Since this value is only subjected to small variations on account of the precession, I have during both periods made use of the value of L reduced to 1898.0 or $L = 74^\circ.3$.

Employing the equation (31) for the computation of p and q , the mean values of $\sin u_0$ and $\cos u_0$ for the different parts of the year must first be cal-

culated. These mean values as well as the values of u_0 are given in table XXVIII. The normal equations derived from (31) are of the following form:

$$(32) \quad \begin{cases} p \Sigma \sin^2 u_0 + q \Sigma \sin u_0 \cos u_0 = \Sigma k y_0 \sin u_0, \\ p \Sigma \sin u_0 \cos u_0 + q \Sigma \cos^2 u_0 = \Sigma k y_0 \cos u_0. \end{cases}$$

Here $\sin u_0$ and $\cos u_0$ signify the means of these functions given in table XXVIII. Since

$$\begin{aligned} \Sigma \sin^2 u_0 &= \Sigma \cos^2 u_0 = 4.840, \\ \Sigma \sin u_0 \cos u_0 &= 0, \end{aligned}$$

the normal equations will be reduced to

$$(33) \quad \begin{cases} 4.840p = \Sigma k y_0 \sin u_0, \\ 4.840q = \Sigma k y_0 \cos u_0. \end{cases}$$

In using the values of y_0 , given in table XXVII, corrected for the mean of the latitude drift in the hemisphere in question, we obtain the following values for p and q , or what is the same, $\Delta L \sin I$ and ΔI .

First Period (1886—1897):

$$\text{Northern Hemisphere: } \Delta L \sin I = + 2'.75 \pm 1'.59; \quad \Delta I = - 10'.91 \pm 1'.59;$$

$$\text{Southern Hemisphere: } \Delta L \sin I = + 5'.53 \pm 2'.70; \quad \Delta I = - 11'.00 \pm 2'.70;$$

$$\text{Mean } \Delta L \sin I = + 4'.14 \pm 1'.57, \quad \Delta I = - 10'.96 \pm 1'.57.$$

Second Period (1898—1909):

$$\text{Northern Hemisphere: } \Delta L \sin I = + 3'.56 \pm 2'.58; \quad \Delta I = - 2'.78 \pm 2'.58;$$

$$\text{Southern Hemisphere: } \Delta L \sin I = + 0'.91 \pm 3'.10; \quad \Delta I = - 0'.37 \pm 3'.10;$$

$$\text{Mean } \Delta L \sin I = + 2'.24 \pm 2'.02, \quad \Delta I = - 1'.58 \pm 2'.02.$$

These values differ considerably from each other, an example of the difficulty of accurately determining the position of the sun's axis. Whether the variations in the values of $\Delta L \sin I$ are actual or only accidental is rather difficult to determine. It seems, however, as if the variations were accidental, especially as the two hemispheres during both periods give considerably deviating values. Even the great mean errors seem to indicate the variations to be accidental. As regards ΔI it seems, however, as if this quantity had been increasing in no small degree from the one period to the other. That this circumstance is not accidental appears from a comparison with the results obtained by DYSON and MAUNDER, which results are brought together in table XXV, borrowed from their paper. In computing from this table the mean values of $\Delta L \sin I$ and ΔI during the periods 1886—1897 and 1898—1909, without regard to the weights, we obtain

$$\text{First Period (1886—1897): } \Delta L \sin I = + 1'.8 \pm 1'.6; \quad \Delta I = - 8'.0 \pm 1'.4;$$

$$\text{Second Period (1898—1909): } \Delta L \sin I = - 1'.2 \pm 2'.1; \quad \Delta I = - 1'.5 \pm 2'.3.$$

Thus it really seems as if the angle between the sun's equator and the ecliptic had been somewhat diminishing during the time embraced in the investigation.

That the obtained values of the corrections of the inclination as well as of the node must be attended by great mean errors appears from table XXVII. The mean errors of the values of y_0 , computed from the entire material, are given in the last column of this table. These mean errors are all of the size of $0'.030$, and are thus rather great in proportion to the variations the latitude motions are subject to, during the course of the year. The corresponding mean errors of the values of y_0 , computed from the motions in one hemisphere only, during one of the periods in question, will thus be of the size of about $0'.060$. These values of y_0 being the direct basis of the computation of $\Delta L \sin I$ and ΔI , one must, as above mentioned, positively expect also these corrections to be attended by considerable mean errors.

CHAPTER IV

The Dispersion of the Motions for different Spot-sizes.

We shall now more closely investigate the dispersion of the motions for different spot-sizes. Already in discussing the positive excess of the longitude as well as of the latitude motions, we pointed out the evident connection existing between the dispersion and spot-area. Since the dispersion might be considered as a measure of the stability of the spots during the motion, my first idea was to suppose a smaller dispersion for the greater spots. This also proved to be the case. Then we did not take the following matter into consideration. Since the co-ordinates of the great spots are determined with considerably greater uncertainty than the co-ordinates of the small ones, this circumstance should evidently contribute to a greater dispersion of the motions of the larger spots than of the smaller.

Now we shall first compute the dispersion of the motions for different spot-sizes. In treating the question of the positive excess of the latitude motions, the distribution of the motions for different spot-sizes was also investigated. The result of these computations for the different hemispheres during both periods are brought together in table XXIX. In this table the dispersions of the longitude motions are also given. Since the mean values of the motions in longitude vary with the latitude, it may at first sight seem appropriate to combine different latitudes in computing these dispersions. The dispersions σ_x given in table XXIX are, however, obtained in displacing the distributions in such a way that the means coincide. These dispersions have thus been computed from the values of table XVI, by means of the well known formula

$$n\sigma_x^2 = \sum n\sigma^2,$$

where the terms in the right membrum depending on the differences between the means are omitted.

Looking somewhat more closely at the table, we find a pervading diminishing of the dispersion with increasing spot-area, during the two periods the investigation embraces. Combining the entire material, we obtain the values of the four last

TABLE XXIX.
The Dispersion of the Motions for different Spot-sizes.

Logarithm of Spot-area	First Period (1886—1897)						Second Period (1898—1909)						1886—1909			
	Northern Hemisphere			Southern Hemisphere			Northern Hemisphere			Southern Hemisphere						
	n	σ_x	σ_y	n	σ_x	σ_y	n	σ_x	σ_y	n	σ_x	σ_y	n	σ_x	$\bar{\sigma}_y$	$\sigma_y : \sigma_x$
$0.80 < w < 1.20$	371	2.813	1.523	499	2.926	1.426	342	3.168	1.626	342	3.123	1.530	1554	2.999	1.518	0.506
$1.20 < w < 1.60$	658	3.226	1.495	895	2.887	1.403	628	2.976	1.604	680	2.838	1.486	2861	2.997	1.490	0.501
$1.60 < w < 2.00$	883	2.473	1.214	982	2.439	1.204	779	2.570	1.318	906	2.609	1.306	3550	2.519	1.239	0.500
$2.00 < w < 2.30$	1108	2.048	1.053	1321	1.890	0.959	998	1.875	1.090	997	1.887	1.058	4424	1.927	1.056	0.538
$2.40 < w < 2.80$	819	1.735	0.982	887	1.661	0.894	659	1.869	1.063	524	1.632	1.011	2899	1.727	0.967	0.560
$2.80 < w < 3.20$	190	1.841	0.852	296	1.604	0.948	185	1.489	1.130	124	1.738	0.892	865	1.696	0.958	0.565

columns. From these values the intimate connection between the dispersion and spot-area is still more obvious.

Since σ_x is about double the size of σ_y , we see that the dispersions in longitude, as well as in latitude, are approximately the same function of the spot-area. In fig. 7 the values of the dispersions, computed from the entire material of observation, are graphically represented.

Computing the quotient $\sigma_y : \sigma_x$ for the different spot-sizes, we obtain the values given in the last column of table XXIX. It seems as if this quotient for the larger spots should have a greater value than for the smaller ones. Since the dispersion for spots of a certain size may be regarded as a measure of the currents in the photosphere, one might possibly infer that the quotient between the sizes of the currents perpendicular and parallel to the equator, is greater for the larger than for the smaller spots. This might be supposed to be due to $\sigma_y : \sigma_x$, as well as the spot-area, varying with the latitude, or one might also suppose that the larger and smaller spots generally occur at different depths of the photosphere. As far as I can find, nothing seems to prevent the state of the currents possibly being different at different depths. We saw, however, page 49, that there did not exist any marked connection between the quotient $\sigma_y : \sigma_x$ and the latitude, wherefore the first assumption does not seem very probable. Should it, however, be the case that the larger spots generally occur at higher level than the smaller, this might possibly explain the fact, stated on page 43, that the larger spots give a shorter rotation period than the smaller ones, which fact would thus indicate a greater angular velocity of the outer layers of the sun than that of the lower.

The observed dispersion of the motions evidently arises from two sources:

1. *Actual proper motions of the spots;*
2. *Errors in the measurements of the photographs.*

If we call the dispersions arising from these two sources, σ_1 and σ_2 respectively, and the observed dispersion σ , we may put

$$(34) \quad \sigma^2 = \sigma_1^2 + \sigma_2^2.$$

Of course, it may now be expected that σ_1 will diminish, while, on the other hand, σ_2 must undoubtedly increase with increasing spot-area. A closer examination now shows that σ diminishes in no small degree as the spot-area increases. Hence we

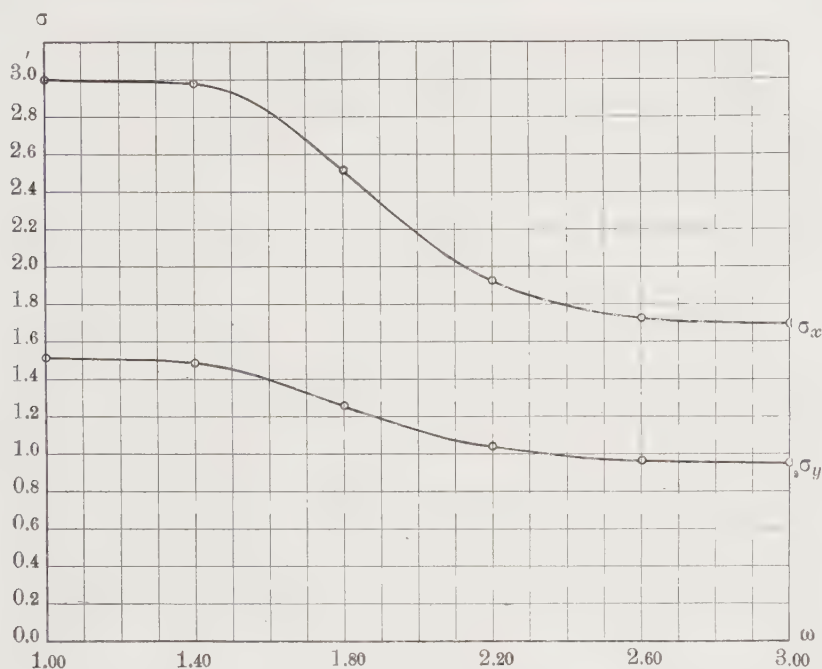


Fig. 7. The Dispersion in Longitude and Latitude as Function of the Spot-area.

may conclude that the part of the dispersion arising from the errors of observation is rather inconsiderable, when compared with that part of the dispersion which is caused by the actual motions of the spots.

I had thought to examine the relation between σ_1 and σ_2 in some way or other. This proves, however, to be rather a difficult task. A method that might possibly be practicable is the following. Suppose the motions of the spots to be determined not as we have done, from the co-ordinates on two consecutive days, but from greater intervals. In this way one might possibly assume σ_2 to be reduced to a small quantity, and the observed dispersion might be considered as arising only from the proper motions of the spots. But the dispersion obtained in this way is evidently not quite of the same size as σ_1 . If this were the case, σ_2 might be obtained directly

from the equation (34). It would, however, involve very long and arduous work to obtain a result in this way.

Another method, which if practicable, would more easily give a result, is the following. We assume a probable hypothesis of the connection between σ_1 and σ_2 on one side, and the spot-area on the other, and afterwards we try to determine the constants. Thus, by way of trial, I have assumed the dispersion arising from the proper motion of the spots to follow a law of the form

$$(35) \quad \sigma_1 = \frac{k_1}{\sqrt{\omega}},$$

analogous to MAXWELL's law of the distribution of the velocities of molecules, where

$$m\sigma^2 = \text{a constant},$$

m being the mass of a molecule. The dispersion arising from errors in the measurements I suppose to be proportional to the section of the spot, thus

$$(36) \quad \sigma_2 = k_2 10^{\frac{1}{2}\omega}.$$

Hence it follows, that the observed dispersion σ might be expressed by

$$(37) \quad \sigma^2 = \frac{k_1^2}{\omega} + k_2^2 10^{\omega}.$$

Now the above, as well as other similar hypotheses, do not exactly agree with the graphical representation, given in fig. 7, for which reason I do not here state the calculations made. That it does not seem quite possible to give σ_2 as a simple function of the spot-area is evident from the fact of the large spots often consisting of a great number of small components. Since, however, owing to reasons previously stated, it would seem as if the portion of the dispersion arising from errors in the measurements, forms a fairly inconsiderable part of the observed dispersion, I do not enter into any determination of σ_1 and σ_2 . Besides, the equation (35) gives an approximate representation of the dispersion as a function of the spot-area. The curves given in fig. 7 are thus drawn through the observed values of dispersion, without the aid of analytical expressions.

That this intimate connection existing between the dispersion and the spot-area has a simple mechanical explanation seems most probable.

CHAPTER V.

Correlation between the Longitude and Latitude Motions of the Sun-spots.

In the preceding chapters we have investigated the motions in longitude and latitude respectively. Now we shall examine whether any connection exists between these motions. When the material of observation was first divided according to $\Delta\lambda \cos \beta (=x)$ and $\Delta\beta (=y)$, this distribution was given in the form of correlation-tables between x and y . Such tables were obtained for each latitude-zone, 2,5 degrees broad, during each year of both periods. These original tables, amounting to 438, I have given on pages 66—71, properly combined. In order to obtain a better survey of the distribution, I have used a class-breadth of 1.0 also in the case of latitude motion.

Even a cursory examination of these correlation-tables shows that the line of symmetry of the frequency-surface of the motions within all zones in the northern hemisphere has a different position from the line of symmetry in the southern zones. From the tables it appears that in the former case, the greater of the principal axes passes through the second and fourth quadrants, but through the first and third in the latter.

Hence we see that a connection does exist between the longitude and latitude motions. The degree of such a connection is usually expressed by a number, the *coefficient of correlation* r , the value of which cannot exceed ± 1 . If $r = 0$, the variables x and y are independent, but if $r = \pm 1$, a certain value of y corresponds to a certain value of x , and vice versa. For further information, the reader is referred to works on mathematical statistics¹.

Designating the relative moments about the mean by

$$\nu_{20}, \nu_{11}, \nu_{02}$$

we have

$$(38) \quad \left\{ \begin{array}{l} \sigma_x^2 = \nu_{20}; \\ \sigma_y^2 = \nu_{02}; \\ r \cdot \sigma_x \sigma_y = \nu_{11}. \end{array} \right.$$

¹ For example G. UDNY YULE: An Introduction to the Theory of Statistics. London 1912.
Lunds Universitets Årsskrift. N. F. Afd. 2. Bd 10.

TABLE XXX.

Correlation between the Motions in Longitude and Latitude 1886–1909.

Zone N_1 .Class-interval in $\Delta\lambda \cos \beta (=x)$ and in $\Delta\beta (=y) = 1'.0$.
Provisory Mean in x and $y = -0'.5$.

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	—	—	—	—	1	—	—	—	—	3	—	—	—	—	—	—	—	—	—	4
+4	—	—	—	—	—	—	—	—	—	—	—	2	2	2	—	—	—	1	1	—	8
+3	—	—	—	—	1	1	1	—	1	1	2	3	4	1	—	1	1	2	—	—	19
+2	—	—	—	2	2	2	1	4	2	7	12	14	7	6	5	3	—	2	—	—	69
+1	—	—	—	—	1	2	2	7	14	30	72	60	27	12	3	4	1	2	—	—	238
0	—	—	—	—	2	1	1	5	7	13	46	33	25	12	8	2	2	1	—	1	159
-1	—	—	—	—	—	1	4	4	3	7	13	7	8	4	4	3	—	—	1	—	59
-2	—	—	—	—	—	—	—	2	—	2	3	2	1	3	—	1	1	1	1	—	17
-3	—	—	—	—	—	—	—	—	1	1	2	—	1	1	—	—	—	—	—	—	6
-4	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	2
Totals	—	—	—	2	6	9	9	22	28	61	153	121	75	41	21	14	6	9	3	1	581

Zone S_1 .

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+4	—	—	—	—	—	—	—	1	—	1	—	1	—	1	—	—	—	2	—	—	6
+3	—	—	—	—	—	1	1	2	3	1	1	2	2	2	2	1	1	2	—	—	21
+2	—	1	—	—	—	1	—	5	2	5	19	16	9	2	3	2	3	2	1	1	72
+1	—	1	—	1	—	2	3	5	5	23	87	72	28	14	8	5	2	1	2	—	259
0	—	1	—	—	1	1	1	6	10	23	77	59	15	12	3	3	3	2	1	—	218
-1	—	—	1	—	—	—	2	—	3	4	13	9	8	8	3	—	1	—	—	—	52
-2	1	—	—	—	2	1	1	5	4	3	—	2	3	—	1	—	—	—	—	—	23
-3	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
-4	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	2
Totals	1	4	1	1	3	6	8	25	27	60	197	161	65	40	21	11	10	9	4	1	655

TABLE XXX. Cont.

Correlation between the Motions in Longitude and Latitude 1886–1909.

Zone N_2 .

Class-interval in $\Delta\lambda \cos \beta (=x)$ and in $\Delta\beta (=y) = 1'.0$.
 Provisory Mean in x and in $y = -0'.5$.

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	1	—	—	—	—	—	—	—	1	1	—	1	1	—	—	3	—	—	—	—	8
+4	—	—	—	—	—	—	—	1	—	3	—	7	1	—	2	1	1	—	—	—	16
+3	—	1	—	1	3	3	5	4	6	7	17	4	16	6	6	—	1	—	—	—	80
+2	—	2	1	—	1	7	8	12	15	35	38	36	27	11	7	5	4	2	—	1	212
+1	1	1	1	3	2	4	13	27	53	99	208	140	73	35	21	9	5	1	1	1	698
0	—	—	1	1	1	6	6	19	27	78	147	118	63	32	15	3	10	4	1	2	534
-1	1	—	1	2	3	3	2	8	12	22	39	29	27	16	22	6	4	4	4	—	205
-2	—	—	—	—	—	—	—	1	2	7	20	5	9	2	3	1	3	3	1	—	57
-3	—	—	1	1	—	—	1	2	3	—	3	5	1	2	5	1	1	—	—	2	28
-4	—	—	1	—	—	1	2	—	—	2	1	—	1	1	—	—	—	—	—	1	10
Totals	3	4	6	8	10	24	37	74	119	254	473	345	219	105	81	29	29	14	7	7	1848

Zone S_2 .

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	—	—	1	1	—	—	—	1	—	1	—	—	1	1	—	—	2	—	—	8
+4	—	—	—	—	—	2	1	1	—	4	—	1	3	1	—	—	—	1	1	—	15
+3	—	—	1	1	1	—	1	2	5	7	8	10	6	6	4	1	1	1	3	2	60
+2	—	—	—	1	—	—	3	8	9	21	49	49	29	21	12	7	5	3	3	1	221
+1	—	—	2	2	2	5	11	24	38	131	252	163	72	57	30	15	5	2	2	1	814
0	—	—	—	1	2	5	3	10	27	81	194	119	68	43	16	5	5	1	2	1	583
-1	—	—	—	—	1	3	6	10	20	24	50	29	32	12	8	6	2	—	—	—	203
-2	—	—	—	1	2	4	6	—	9	5	11	10	4	9	3	—	1	—	2	—	67
-3	1	—	1	—	—	—	2	2	1	1	—	1	3	1	—	—	—	—	—	1	14
-4	1	—	—	1	—	—	2	—	—	2	1	1	1	—	—	1	1	—	—	—	11
Totals	2	—	4	8	9	19	35	57	110	276	566	383	218	151	74	35	20	10	13	6	1996

TABLE XXX. Cont.

Correlation between the Motions in Longitude and Latitude 1886–1909.

Zone N_8 .

Class-interval in $\Delta\lambda \cos \beta(=x)$ and in $\Delta\beta(=y) = 1'.0$.
Provisory Mean in x and $y = -0'.5$.

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	—	1	—	1	—	—	2	4	1	2	—	2	—	1	—	1	—	—	—	15
+4	—	—	—	—	3	4	1	4	3	3	4	5	4	1	—	4	—	—	—	—	36
+3	1	—	—	1	1	1	4	8	13	10	4	15	9	4	2	3	—	2	—	—	78
+2	—	—	2	3	—	5	12	17	23	49	64	66	30	17	9	4	1	4	1	—	307
+1	1	1	2	2	2	11	7	30	64	190	255	186	68	50	26	12	11	7	4	—	929
0	—	—	1	3	3	8	7	24	74	143	217	149	70	41	21	12	10	4	1	—	788
-1	1	1	—	1	4	1	2	5	24	36	54	50	30	19	13	10	7	3	3	—	264
-2	1	—	1	1	—	1	2	4	7	7	9	8	13	3	7	2	1	4	1	1	73
-3	—	—	—	—	—	1	1	2	1	1	4	4	4	1	4	1	1	—	—	2	27
-4	—	—	—	—	—	—	—	—	1	1	—	—	1	3	1	—	—	—	—	—	7
Totals	4	2	7	11	14	32	36	96	214	441	613	483	231	139	84	48	32	24	10	3	2524

Zone S_3 .

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	1	—	—	—	—	2	—	3	1	—	1	1	—	—	—	4	3	—	—	16
+4	—	—	1	—	—	1	—	1	1	1	—	5	2	3	2	4	1	1	1	—	24
+3	—	1	—	2	1	3	—	6	4	5	5	8	7	7	3	2	4	—	3	2	63
+2	—	—	2	5	5	2	4	12	24	42	70	62	37	21	14	13	8	8	1	1	331
+1	1	1	2	1	3	2	13	23	71	182	271	172	100	59	25	16	5	3	3	—	953
0	1	—	2	1	8	6	9	27	73	135	209	131	72	31	17	7	9	2	1	—	741
-1	—	—	2	5	4	4	18	14	31	58	51	44	30	19	6	9	3	1	1	—	300
-2	—	—	2	3	3	5	7	6	12	12	14	11	6	1	—	—	1	3	—	—	86
-3	—	1	—	—	—	1	2	—	2	3	6	3	3	—	2	—	—	1	—	—	24
-4	1	—	—	1	—	1	—	—	4	1	—	—	1	—	1	—	—	—	—	—	10
Totals	3	4	11	18	24	25	55	89	225	440	626	437	259	141	70	51	35	22	10	3	2548

TABLE XXX. Cont

Correlation between the Motions in Longitude and Latitude 1886—1909.

Zone N₄.

Class-interval in $\Delta\lambda \cos \beta (=x)$ and in $\Delta\beta (=y) = 1'.0$.
 Provisory Mean in x and $y = -0'.5$.

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	1	—	—	—	3	2	—	—	2	5	1	—	—	—	—	—	—	—	—	14
+4	—	1	1	—	2	2	3	4	2	3	3	6	—	2	1	—	—	—	—	—	30
+3	—	—	1	1	3	3	5	6	3	11	6	3	4	1	2	1	—	1	—	—	51
+2	—	—	—	2	2	8	4	16	38	36	31	28	15	8	9	5	1	2	1	—	206
+1	—	1	3	5	3	4	13	27	78	130	180	80	33	32	11	11	8	4	2	1	626
0	—	1	2	2	2	3	6	19	39	101	127	115	46	21	17	8	—	4	3	—	516
-1	—	—	—	3	2	5	3	8	12	30	45	32	21	19	7	9	3	4	3	—	206
-2	—	—	—	—	2	1	1	2	2	5	9	6	5	5	6	4	2	3	—	—	53
-3	1	—	—	1	—	—	1	—	1	—	2	—	3	3	3	1	—	—	2	1	19
-4	—	—	—	—	—	—	—	—	1	1	—	—	1	1	—	—	—	—	—	—	4
Totals	1	4	7	14	16	29	38	82	176	319	408	271	128	92	56	39	14	18	11	2	1725

Zone S₄.

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	—	—	—	—	—	1	—	—	—	—	2	—	2	1	—	—	—	—	—	6
+4	—	—	—	1	—	—	—	2	2	1	2	2	3	—	—	1	1	—	—	—	15
+3	—	—	1	—	—	1	1	2	4	5	9	14	11	4	10	1	2	3	1	1	70
+2	1	1	—	1	4	4	4	10	25	52	61	49	25	21	15	9	3	4	2	—	291
+1	1	1	—	1	1	10	13	27	82	157	231	128	63	39	15	23	1	2	1	—	796
0	2	1	—	1	4	6	12	28	75	147	178	98	50	24	13	8	6	5	—	—	658
-1	—	1	1	3	2	7	15	19	26	47	40	31	21	13	4	4	2	1	—	1	238
-2	—	1	—	1	4	1	2	7	9	7	17	9	8	2	4	3	4	1	1	—	81
-3	—	2	—	—	—	1	1	2	—	4	3	1	4	1	2	—	—	1	—	—	22
-4	1	2	—	—	1	1	2	—	3	—	—	—	1	—	—	2	—	—	—	—	13
Totals	5	9	2	8	16	31	51	97	226	420	541	334	186	106	64	51	19	17	5	2	2190

TABLE XXX. Cont.

Correlation between the Motions in Longitude and Latitude 1886–1909.

Zone N_6 .Class-interval in $\Delta\lambda \cos \beta (=x)$ and in $\Delta\beta (=y) = 1'.0$.Provisory Mean in x and $y = -0'.5$.

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	—	—	—	2	—	—	2	2	—	1	1	—	—	—	—	—	—	—	—	8
+4	—	—	—	—	—	—	3	—	1	1	—	2	—	—	2	—	1	—	—	—	10
+3	—	—	—	1	—	2	1	7	5	9	3	8	1	1	—	—	2	—	—	—	40
+2	—	1	—	—	3	3	12	15	16	26	23	8	7	5	4	3	2	—	—	—	128
+1	—	1	1	1	3	3	3	19	35	83	83	37	18	11	5	2	1	2	2	—	310
0	—	—	—	1	1	3	6	9	39	70	53	38	20	14	6	6	3	—	1	—	270
-1	—	—	—	—	—	2	1	2	11	19	26	11	11	8	2	4	4	1	1	1	104
-2	—	—	—	—	—	—	—	2	6	5	4	6	6	2	1	—	4	1	—	—	37
-3	—	—	—	—	1	—	1	1	—	2	1	2	—	—	1	1	2	—	1	—	13
-4	—	—	—	—	—	1	1	—	1	—	2	3	2	1	—	2	—	2	—	—	15
Totals	—	2	1	3	10	14	28	57	116	215	196	116	65	42	21	18	19	6	5	1	935

Zone S_6 .

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	1	1	1	—	—	5
+4	—	1	—	—	1	—	2	—	1	—	1	1	—	3	—	1	—	—	—	—	11
+3	—	—	—	1	1	2	3	3	5	1	7	6	4	2	1	1	3	2	—	—	42
+2	—	—	—	—	1	2	2	8	10	21	30	16	13	8	7	3	3	1	2	—	127
+1	—	1	—	4	1	6	6	17	38	80	98	62	43	13	6	5	2	1	2	—	385
0	—	—	2	1	2	5	6	19	42	66	72	38	25	12	5	4	3	—	—	—	302
-1	—	—	—	5	2	3	3	14	26	19	25	16	7	5	2	1	2	1	—	1	132
-2	—	—	—	4	1	—	2	5	5	7	8	1	4	1	1	1	—	—	—	—	40
-3	—	—	—	1	2	1	3	2	5	—	1	—	—	1	1	—	—	—	—	—	17
-4	1	—	1	—	—	—	—	—	1	1	—	—	1	—	—	—	—	—	—	—	5
Totals	1	2	3	16	11	19	27	68	134	195	242	141	97	45	23	17	14	6	4	1	1066

TABLE XXX. Cont.

Correlation between the Motions in Longitude and Latitude 1886—1909.

Zone N_6 .

Class-interval in $\Delta\lambda \cos \beta(=x)$ and in $\Delta\beta(=y) = 1'.0$.
Provisory Mean in x and $y = -0'.5$.

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
+4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
+3	—	—	—	—	—	—	—	1	1	—	1	2	—	—	—	1	—	—	—	—	6
+2	1	—	—	1	2	1	3	3	2	6	5	4	—	—	—	—	—	—	—	1	29
+1	—	—	—	2	—	2	2	4	23	27	4	8	4	1	2	1	—	—	—	—	90
0	—	—	—	—	1	1	1	3	8	14	15	6	6	—	1	1	—	1	—	—	58
-1	—	—	—	—	1	—	—	—	3	3	8	3	1	2	1	—	—	1	1	—	24
-2	—	—	—	—	—	—	—	—	1	—	3	5	3	1	—	—	—	—	—	—	13
-3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	2
-4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	1	—	—	3	4	4	6	11	38	50	46	28	15	4	5	3	1	2	1	2	224

Zone S_6 .

$\begin{smallmatrix} x \\ y \end{smallmatrix}$	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10	Totals
+5	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	3
+4	—	—	—	—	—	—	1	1	1	—	—	1	—	—	—	—	—	—	—	—	4
+3	—	—	—	—	1	—	—	1	1	—	3	5	3	3	2	2	—	—	—	—	21
+2	—	—	—	—	—	—	2	3	6	2	10	10	7	2	2	2	—	—	—	1	47
+1	—	—	—	—	1	2	4	9	11	25	29	6	5	3	1	1	—	—	—	1	98
0	—	1	—	1	2	—	5	7	17	21	20	10	5	3	2	1	1	—	—	—	96
-1	—	—	—	—	—	—	1	5	6	13	2	4	1	—	—	—	—	—	—	—	32
-2	—	1	—	—	—	4	2	—	2	4	1	3	1	—	—	—	—	—	—	—	18
-3	—	—	—	1	—	—	—	2	—	—	1	—	—	—	—	—	—	—	—	—	4
-4	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	—	2	—	2	4	6	15	28	44	65	67	40	22	11	8	7	1	—	—	2	324

Choosing the mean of the frequency-surface as the origin of a system of co-ordinates, with the axes parallel and perpendicular to the equator, the curves of equal frequency will be represented by the ellipses

$$\frac{x^2}{\nu_{20}} - \frac{2\nu_{11}}{\nu_{20}\nu_{02}}xy + \frac{y^2}{\nu_{02}} = \text{constant}.$$

Here x and y designate the co-ordinates in this system. In order to make the axes of our system of co-ordinates coincide with the principal axes, the system must be turned an angle φ , which may be easily computed in the following way. Suppose X and Y to be co-ordinates referred to the principal axes, we have

$$(39) \quad \begin{cases} X = x \cos \varphi + y \sin \varphi, \\ Y = y \cos \varphi - x \sin \varphi. \end{cases}$$

Since X and Y are uncorrelated,

$$\Sigma XY = 0.$$

Hence it follows, by multiplying the equations (39) and summing,

$$0 = (-\nu_{20} + \nu_{02}) \sin 2\varphi + 2\nu_{11} \cos 2\varphi,$$

or

$$(40) \quad \tan 2\varphi = \frac{2\nu_{11}}{\nu_{20} - \nu_{02}} = \frac{2r\sigma_x\sigma_y}{\sigma_x^2 - \sigma_y^2}.$$

Since

$$\sigma_x > \sigma_y,$$

we have

$$0 < \varphi < +90^\circ, \quad \text{if} \quad r > 0,$$

and

$$0 > \varphi > -90^\circ, \quad \text{if} \quad r < 0.$$

The dispersions Σ_x and Σ_y about these new axes, the principal axes, are easily obtained from the equations

$$(41) \quad \begin{cases} \Sigma_x^2 + \Sigma_y^2 = \sigma_x^2 + \sigma_y^2, \\ \Sigma_x \Sigma_y = \sigma_x \sigma_y \sqrt{1 - r^2}. \end{cases}$$

Since, in the present case, r is a small quantity, the new dispersions will differ inconsiderably from σ_x and σ_y . The results of the numerical computations are given in table XXXI. From this table the fact of the correlation coefficient r being negative in the northern hemisphere, and positive in the southern, is clearly proved. The angle between the line of symmetry of the frequency-surface and the equator, seems, upon an average, to increase with increasing latitude. These two facts are evidently due to the different angular velocities of the sun at different latitudes. Hence, the trade-winds on the earth, have to some extent their correspondence on the sun though their directions are inverted.

In this connection we will devote some remarks to the following problem: *Is it possible to divide our correlation-surfaces into two normal frequency-surfaces?* Professor CHARLIER has in his *Stellar Statistics II*, page 93, completely solved the problem of dividing a given correlation-surface into two *spherical* components.

Thus, if

$$F(x, y)$$

is the correlation-function of the motions of the sun-spots, CHARLIER puts

$$NF(x, y) = N_1 f_1(x, y) + N_2 f_2(x, y),$$

where $f_1(x, y)$ and $f_2(x, y)$, determined from the relations

$$f_1(x, y) = \frac{1}{\alpha_1^2 \cdot 2\pi} e^{-\frac{(x-m_1)^2 + (y-m_1)^2}{2\alpha_1^2}},$$

$$f_2(x, y) = \frac{1}{\alpha_2^2 \cdot 2\pi} e^{-\frac{(x-m_2)^2 + (y-m_2)^2}{2\alpha_2^2}},$$

designate the normal frequency-functions of the two components. The problem then is to determine

$$N_1, \alpha_1, m_1, n_1,$$

$$N_2, \alpha_2, m_2, n_2.$$

When the moments of $F(x, y)$ are known, CHARLIER shows how these quantities may be determined. It appears that the problem in question leads to numerical com-

TABLE XXXI.
Coefficient of Correlation and Line of Symmetry.

Lati- tude- Zones	<i>n</i>	<i>v</i> ₂₀	<i>v</i> ₀₂	<i>v</i> ₁₁	<i>r</i>	<i>ε</i> (<i>r</i>)	<i>φ</i>	<i>σ</i> _{<i>x</i>}	<i>Σ</i> _{<i>x</i>}	<i>σ</i> _{<i>y</i>}	<i>Σ</i> _{<i>y</i>}
<i>N</i> ₀	224	6.580	1.481	−0.759	−0.243	0.063	− 8.3	2.567	2.588	1.217	1.171
<i>N</i> ₅	935	6.007	2.217	−0.796	−0.228	0.081	−10.9	2.451	2.462	1.424	1.369
<i>N</i> ₄	1725	6.355	1.633	−0.700	−0.217	0.023	− 8.3	2.521	2.541	1.278	1.238
<i>N</i> ₃	2524	5.478	1.528	−0.341	−0.118	0.020	− 4.9	2.341	2.347	1.236	1.224
<i>N</i> ₂	1848	5.940	1.623	−0.299	−0.096	0.021	− 3.9	2.437	2.441	1.274	1.266
<i>N</i> ₁	581	5.558	1.541	−0.013	−0.004	0.041	− 0.2	2.358	2.358	1.241	1.241
<i>S</i> ₁	655	5.529	1.233	+0.432	+0.165	0.038	+ 5.7	2.351	2.360	1.110	1.090
<i>S</i> ₂	1996	5.063	1.418	+0.295	+0.110	0.022	+ 4.6	2.250	2.255	1.191	1.181
<i>S</i> ₃	2548	5.885	1.521	+0.523	+0.175	0.019	+ 6.7	2.426	2.438	1.235	1.210
<i>S</i> ₄	2190	5.682	1.522	+0.428	+0.145	0.021	+ 5.8	2.384	2.362	1.234	1.216
<i>S</i> ₅	1066	6.256	1.700	+0.674	+0.207	0.029	+ 8.2	2.501	2.521	1.304	1.266
<i>S</i> ₆	324	5.927	1.989	+1.093	+0.318	0.050	+14.5	2.435	2.462	1.410	1.396

putations considerably more simple than the problem of dividing a given frequency-curve into its two components, which problem, as is shown by PEARSON, generally leads to an equation of the ninth degree (The nomic of PEARSON).

It now being a question of the motions of the sun-spots, there is nothing giving countenance to the possibility of dividing the correlation-surfaces into *spherical* components, since, according to the previous investigations, the dispersions of the longitude motions are about double the size of the dispersions of the motions in latitude. A trial with the formulæ developed by CHARLIER gave the following improbable

values of the quotient $N_1:N_2$, where N_1 and N_2 are the numbers of spots belonging to the different components of the frequency-surface.

	$\frac{N_1}{N_2}$		$\frac{N_1}{N_2}$
Zone N_1	1.37	Zone S_1	0.99
» N_2	2.12	» S_2	2.93
» N_3	0.73	» S_3	0.97
» N_4	0.67	» S_4	0.72
» N_5	0.74	» S_5	1.23
» N_6	0.71	» S_6	1.30.

On the other hand, several circumstances seem to indicate that the dispersions in longitude are the same in both streams and likewise the dispersions in latitude. Thus, we will assume that our correlation-surfaces may be divided into two normal frequency-surfaces of the form

$$(42) \quad \begin{aligned} f_1(x, y) &= \frac{1}{\alpha_1 \alpha_2 \cdot 2\pi} e^{-\frac{(x-m_1)^2}{2\alpha_1^2} - \frac{(y-n_1)^2}{2\alpha_2^2}}, \\ f_2(x, y) &= \frac{1}{\alpha_1 \alpha_2 \cdot 2\pi} e^{-\frac{(x-m_2)^2}{2\alpha_1^2} - \frac{(y-n_2)^2}{2\alpha_2^2}}. \end{aligned}$$

Further we suppose m and n to be the means of $F(x, y)$, and the moment ν_{ij} of $F(x, y)$ to be known.

In putting

$$NF(x, y) = N_1 f_1(x, y) + N_2 f_2(x, y)$$

according to CHARLIER, multiplying with $x^i y^j$ and integrating, the following equations may be deduced, if the moments are taken about m and n .

$$(43) \quad \begin{aligned} N &= N_1 & + N_2, \\ 0 &= N_1(m_1 - m) & + N_2(m_2 - m), \\ 0 &= N_1(n_1 - n) & + N_2(n_2 - n), \\ N\nu_{20} &= N_1(\alpha_1^2 + (m_1 - m)^2) & + N_2(\alpha_1^2 + (m_2 - m)^2), \\ N\nu_{02} &= N_1(\alpha_2^2 + (n_1 - n)^2) & + N_2(\alpha_2^2 + (n_2 - n)^2), \\ N\nu_{11} &= N_1(m_1 - m)(n_1 - n) & + N_2(m_2 - m)(n_2 - n), \\ N\nu_{30} &= N_1(3\alpha_1^2(m_1 - m) + (m_1 - m)^3) & + N_2(3\alpha_1^2(m_2 - m) + (m_2 - m)^3), \\ N\nu_{03} &= N_1(3\alpha_2^2(n_1 - n) + (n_1 - n)^3) & + N_2(3\alpha_2^2(n_2 - n) + (n_2 - n)^3), \\ N\nu_{40} &= N_1(N_{40} + 6\nu_{20}(m_1 - m)^2 + (m_1 - m)^4) & + N_2(N_{40} + 6\nu_{20}(m_2 - m)^2 + (m_2 - m)^4), \\ N\nu_{04} &= N_1(N_{04} + 6\nu_{02}(n_1 - n)^2 + (n_1 - n)^4) & + N_2(N_{04} + 6\nu_{02}(n_2 - n)^2 + (n_2 - n)^4), \end{aligned}$$

In the last two equations N_{40} and N_{04} designate the fourth moments of the functions $f_1(x, y)$ and $f_2(x, y)$.

Introducing $m_1 - m_2$ and $n_1 - n_2$ in stead of $m_1 - m$, $m_2 - m$, $n_1 - n$ and $n_2 - n$ by means of the relations

$$\begin{aligned}
 m_1 - m &= \frac{N_2}{N} (m_1 - m_2), \\
 m_2 - m &= -\frac{N_1}{N} (m_1 - m_2), \\
 n_1 - n &= \frac{N_2}{N} (n_1 - n_2), \\
 n_2 - n &= -\frac{N_1}{N} (n_1 - n_2),
 \end{aligned}
 \tag{44}$$

the above equations are simplified into the following, where I have given $v_{40} - 3v_{20}^2$ and $v_{04} - 3v_{02}^2$ in stead of v_{40} and v_{04} .

$$\begin{aligned}
 N &= N_1 + N_2, \\
 \sigma_x^2 = v_{20} &= \alpha_1^2 + \frac{N_1 N_2}{NN} (m_1 - m_2)^2, \\
 \sigma_y^2 = v_{02} &= \alpha_2^2 + \frac{N_1 N_2}{NN} (n_1 - n_2)^2, \\
 r\sigma_x\sigma_y = v_{11} &= \frac{N_1 N_2}{NN} (m_1 - m_2)(n_1 - n_2), \\
 -2\sigma_x^3 \cdot S_x = v_{30} &= \frac{N_1 N_2}{NN} \left(\frac{N_2}{N} - \frac{N_1}{N} \right) (m_1 - m_2)^3, \\
 -2\sigma_y^3 \cdot S_y = v_{03} &= \frac{N_1 N_2}{NN} \left(\frac{N_2}{N} - \frac{N_1}{N} \right) (n_1 - n_2)^3, \\
 8\sigma_x^4 \cdot E_x = v_{40} - 3v_{20}^2 &= \frac{N_1 N_2}{NN} \left(1 - 6 \frac{N_1 N_2}{NN} \right) (m_1 - m_2)^4, \\
 8\sigma_y^4 \cdot E_y = v_{04} - 3v_{02}^2 &= \frac{N_1 N_2}{NN} \left(1 - 6 \frac{N_1 N_2}{NN} \right) (n_1 - n_2)^4.
 \end{aligned}
 \tag{45}$$

From these 8 equations, in which I have also expressed the known moments in σ_x , σ_y , S_x , S_y , E_x and E_y , the unknown quantities may be determined.

It is, however, easily seen that such a division is impracticable in the present case. Since E_x and E_y are positive in all zones, it follows that

$$1 - 6 \frac{N_1 N_2}{NN} > 0. \tag{46}$$

This is only possible if any of the quotients $\frac{N_1}{N}$ and $\frac{N_2}{N}$ is smaller than 0.211.

From the last two equations of (45) it follows that

$$\frac{m_1 - m_2}{n_1 - n_2} = \pm \frac{\sigma_x}{\sigma_y} \sqrt[4]{\frac{E_x}{E_y}}. \tag{47}$$

Since

$$(m_1 - m_2)(n_1 - n_2)$$

has the same sign as N_{11} or r , it follows that

$$(48) \quad \frac{m_1 - m_2}{n_1 - n_2} = -\frac{\sigma_x}{\sigma_y} \sqrt[4]{\frac{\overline{E_x}}{\overline{E_y}}} \quad \text{in the northern hemisphere}$$

and

$$(49) \quad \frac{m_1 - m_2}{n_1 - n_2} = +\frac{\sigma_x}{\sigma_y} \sqrt[4]{\frac{\overline{E_x}}{\overline{E_y}}} \quad \text{in the southern.}$$

From the equations of ν_{80} and ν_{03} we deduce

$$(50) \quad \frac{m_1 - m_2}{n_1 - n_2} = \frac{\sigma_x}{\sigma_y} \sqrt[3]{\frac{S_x}{S_y}}.$$

That this expression may not be inconsistent with the preceding expressions, S_x and S_y must necessarily have the same sign in the southern hemisphere and different signs in the northern, *a condition not fulfilled in the present case*. Moreover, that a division of our frequency-surfaces into two normal components of the form given in (42) may be practicable, the relation

$$(51) \quad \sqrt[3]{\frac{S_x}{S_y}} = \sqrt[4]{\frac{\overline{E_x}}{\overline{E_y}}}$$

must exist. This condition can, however, only be successfully used when skewness and excess are not too small. Otherwise the quotients will be very inaccurately determined.

In computing the unknown quantities, N_1 and N_2 are first to be determined. This may be done by means of the expression for ν_{11} combined with the expressions of the third or fourth moment. In the former case we obtain

$$(52) \quad \frac{N_1 N_2}{NN} = \frac{\nu_{11}^3}{\nu_{80} \nu_{03} + 4\nu_{11}^3} = \frac{r^3}{4(S_x S_y + r^3)},$$

in the latter

$$(53) \quad \frac{N_1 N_2}{NN} = \frac{r^2}{6r^2 \pm 8\sqrt{\overline{E_x E_y}}}.$$

In this equation the sign $\pm$ shall be used when E_x and E_y are ≥ 0 or, what is the same, when

$$1 - 6 \frac{N_1 N_2}{NN} \geq 0.$$

Since

$$\frac{N_1}{N} + \frac{N_2}{N} = 1,$$

N_1 and N_2 are determined. Having computed N_1 and N_2 , the other unknown quantities m_1 , m_2 , n_1 , n_2 , α_1 and α_2 are easily obtained. Since we have previously seen that the condition of dividing our frequency-surfaces was not fulfilled, we cannot make a numerical computation of the characteristics of the two frequency-functions (42) in the present case.

CHAPTER VI.

The Distribution of Spots on the Sun's Surface.

A. The Distribution of Spots in the two Hemispheres.

To the question of the distribution of the sun-spots in the two hemispheres I shall here only devote a few remarks, as the manner in which the material of observation is catalogued is not suitable for a closer investigation.

In discussing the motions of the spots, the material of observation within different latitude-zones was divided according to the logarithm of the spot-area ($\log \Delta = \omega$), chiefly because the distribution according to this variable, proved to be almost normal, as also appears from table XXXII, where the characteristics of this distribution are stated. The mean value (ω_0) of the logarithms is evidently nothing but the logarithm of the geometrical mean. The quantities ν_2 , ν_3 and ν_4 designate, as before, the second, third and fourth moments about the mean ω_0 . An expression of the arithmetical mean (Δ_a) of the spot-areas may approximately be deduced in the following manner. Since

$$\Delta_i = 10^{\omega_i} \quad (i = 1, 2, 3 \dots n)$$

we may write

$$(54) \quad \Delta_i = e^{k\omega_i}$$

where

$$k = \frac{1}{10 \log e}.$$

From (54) it follows that

$$\Delta_a = \frac{1}{n} \sum e^{k\omega_i}$$

Designating the deviation of the logarithms from the mean ω_0 by δ , we obtain

$$\begin{aligned} \Delta_a &= \frac{1}{n} \sum e^{k(\omega_0 + \delta_i)} , \\ &= \frac{e^{k\omega_0}}{n} \sum e^{k\delta_i} , \\ &= \frac{e^{k\omega_0}}{n} \left(n + k \sum \delta_i + \frac{k^2}{2} \sum \delta_i^2 + \frac{k^3}{6} \sum \delta_i^3 + \frac{k^4}{24} \sum \delta_i^4 + \dots \right) . \end{aligned}$$

Hence we obtain, designating the geometrical mean of the spot-areas with Δ_0

$$(55) \quad \Delta_a = \Delta_0 \left(1 + \frac{k^2}{2} \nu_2 + \frac{k^3}{3} \nu_3 + \frac{k^4}{4} \nu_4 + \dots \right).$$

Since

$$k = 2.3026$$

we get

$$(56) \quad \Delta_a = \Delta_0 (1 + 2.651\nu_2 + 2.035\nu_3 + 1.171\nu_4 + 0.539\nu_5 + 0.207\nu_6 + 0.068\nu_7 + 0.020\nu_8 + \dots).$$

TABLE XXXII.

Characteristics of the Spot-sizes.

Latitude-zones	n	ω_0	σ	S	E	ν_2	ν_3	ν_4	Δ_a	1000ρ	
I	N_6	176	1.894	0.575	+0.007	-0.053	0.331	-0.003	0.278	108.0	0.336
	N_5	562	1.973	0.614	+0.060	-0.116	0.377	-0.028	0.294	133.6	1.276
	N_4	1081	1.965	0.563	+0.197	-0.072	0.317	-0.070	0.244	124.7	2.220
	N_3	1285	2.017	0.573	+0.204	-0.055	0.323	-0.077	0.276	142.5	2.945
	N_2	709	1.937	0.567	+0.060	-0.065	0.321	-0.022	0.257	117.8	1.324
	N_1	314	1.804	0.501	+0.256	-0.047	0.251	-0.064	0.164	80.8	0.399
	S_1	391	1.907	0.512	+0.254	-0.046	0.294	-0.081	0.227	106.9	0.657
	S_2	1108	1.967	0.584	+0.134	-0.041	0.341	-0.053	0.310	129.3	2.271
	S_3	1367	1.938	0.607	+0.094	-0.072	0.368	-0.042	0.330	123.6	2.718
	S_4	1234	1.918	0.597	+0.090	-0.088	0.356	-0.038	0.291	116.1	2.359
	S_5	621	1.925	0.597	+0.076	-0.103	0.356	-0.032	0.277	117.6	1.242
	S_6	274	1.871	0.663	-0.051	-0.073	0.440	+0.030	0.466	114.2	0.554
II	N_6	48	1.833	0.533	+0.067	+0.056	0.284	-0.020	0.277	92.1	0.078
	N_5	373	1.933	0.558	+0.261	-0.041	0.311	-0.091	0.259	129.8	0.823
	N_4	644	1.877	0.542	+0.097	-0.060	0.294	-0.031	0.217	99.8	1.058
	N_3	1239	1.972	0.618	+0.050	-0.081	0.322	-0.023	0.341	135.3	2.696
	N_2	1139	1.990	0.581	+0.096	-0.068	0.338	-0.033	0.279	135.1	2.439
	N_1	267	1.818	0.540	+0.177	-0.106	0.292	-0.056	0.183	86.2	0.362
	S_1	264	1.886	0.550	+0.046	-0.062	0.302	-0.015	0.229	102.8	0.426
	S_2	888	1.865	0.557	+0.112	-0.065	0.310	-0.039	0.239	98.6	1.388
	S_3	1181	1.939	0.606	+0.082	-0.057	0.367	-0.037	0.343	124.2	2.359
	S_4	956	1.943	0.585	-0.010	-0.092	0.342	+0.004	0.320	123.0	1.936
	S_5	445	1.867	0.559	+0.020	-0.016	0.312	-0.007	0.279	100.3	0.759
	S_6	50	1.812	0.528	+0.112	-0.018	0.279	-0.033	0.222	85.2	0.075

As seen from this equation the numerical convergence is rather slow, except when the dispersion of the logarithms is small. Choosing the following mean values of the moments

$$\nu_2 = +0.330,$$

$$\nu_3 = -0.036,$$

$$\nu_4 = +0.275,$$

we obtain

$$\Delta_a = \Delta_0 (1 + 0.875 - 0.073 + 0.322 + \dots);$$

$$\therefore \Delta_a = 2.12\Delta_0.$$

In the present case the arithmetical mean is thus approximately double the size of the geometrical, the following two terms in the expression for Δ_a only amounting to about -0.02 and $+0.04$.

Although the arithmetical mean cannot be so accurately determined, I have, however, computed it, especially as it will be attended by approximately the same error in the different latitude-zones. In table XXXII the daily spot-density ρ is also given. This density is defined as the quotient between the daily spot-area and the area of the corresponding zone between -60° and $+60^\circ$ from the central meridian.

Such spots as have not existed for at least two consecutive days having been omitted in our catalogue, this will evidently cause an error in the computed spot-densities. The omitted spots being, however, very small, this error will undoubtedly be less than that arising in computing the arithmetical mean from the geometrical.

B. The Distribution of Spots on both Halves of the Sun.

The strange fact that, upon an average, more spots are observed on the eastern half of the sun than on the western, has, it is true, been previously mentioned, but the fact in question does not seem to have been the subject of the attention it possibly deserves.

In table XXXIII I give a summary of all the spots and groups of spots which have been observed on the photographs taken daily at *Greenwich* and other English observatories. The observations are divided according to years and the distance from the central meridian. From the table it is evident that this preponderance of spots on the eastern half cannot be regarded as merely accidental.

That the earth might exercise an influence on the spot-activity of the sun sufficiently large to explain this difference in the frequency of spots, seems scarcely probable, nor can this fact be explained by assuming a certain distribution of the life-duration of the spots. Thus we may suppose that this preponderance of spots is merely apparent, owing to a smaller spot being more easily observed on the eastern half than on the western. This may possibly be due to the conditions of absorption and refraction being different on the two sides of the central meridian on account of the rotation of the sun.

A greater absorption in the solar atmosphere would evidently be the cause of a number of small spots being invisible. Whether such a difference between the atmospheric conditions on the two halves on the sun does actually exist we shall leave undecided. As we shall see later on, there is, however, a circumstance that seems to support the idea of a somewhat different refraction on the two halves of the sun.

Another possibility, suggested by Dr. Block, is that a spot owing to its structure, perhaps caused by different angular velocities of different layers of the sun, may appear unequal on the two sides of the central meridian. The supposition

TABLE XXXIII.
Number of Spots or Groups of Spots at different Distances from the Central Meridian 1886--1909.

	-85°	-75°	-65°	-55°	-45°	-35°	-25°	-15°	-5°	+5°	+15°	+25°	+35°	+45°	+55°	+65°	+75°	+85°	0° to -90°	0° to +90°
1886	14	32	40	50	50	45	50	50	49	48	45	52	49	42	42	41	23	11	380	359
1887	8	26	26	36	34	34	28	31	30	28	30	26	27	27	22	22	15	4	249	201
1888	3	16	13	19	20	15	15	15	15	20	19	11	17	17	19	12	9	3	136	127
1889	3	8	6	12	18	10	11	13	13	16	9	11	14	13	10	10	12	2	94	97
1890	5	6	14	15	18	14	19	17	21	19	23	15	18	15	19	12	7	1	129	129
1891	22	45	74	81	64	94	73	90	85	81	85	79	84	82	72	69	56	19	628	627
1892	33	108	116	134	161	161	153	146	179	166	152	150	155	134	132	116	97	28	1191	1130
1893	43	142	180	183	216	203	203	243	226	214	197	187	190	190	147	143	129	32	1641	1429
1894	51	124	150	171	167	183	186	193	197	208	201	168	185	145	133	126	110	36	1422	1314
1895	26	102	117	140	124	148	150	143	154	149	159	139	139	114	112	96	88	30	1104	1026
1896	11	56	67	72	78	76	78	91	85	101	78	78	79	60	61	60	39	14	614	570
1897	12	50	55	62	59	52	60	63	64	60	69	62	62	47	55	45	38	14	477	452
1898	9	30	42	46	51	54	54	50	56	58	54	56	43	45	47	38	31	6	401	378
1899	3	21	25	33	26	34	30	29	25	26	27	20	22	24	14	17	7	2	226	157
1900	3	8	15	18	14	17	17	22	24	18	24	19	19	12	13	15	9	5	138	134
1901	1	4	6	4	5	4	6	7	2	6	7	3	7	4	4	4	3	1	39	39
1902	1	7	7	9	10	5	8	7	10	9	8	12	9	12	6	5	6	3	64	70
1903	14	35	56	49	53	52	51	60	56	49	48	45	49	46	45	39	29	6	426	356
1904	14	63	77	83	92	92	93	105	104	104	91	93	85	79	69	71	51	15	723	658
1905	25	104	100	114	124	132	124	140	139	127	140	139	111	132	115	98	70	24	1002	956
1906	22	77	77	113	130	120	127	123	141	135	132	134	124	94	115	95	114	29	930	972
1907	36	76	105	98	128	132	130	135	135	123	139	123	129	138	108	109	82	26	975	977
1908	24	73	92	114	119	125	119	118	124	113	108	111	111	89	98	69	77	20	908	796
1909	23	65	81	82	87	100	94	98	98	112	98	93	100	90	79	69	62	18	728	721
1886--1909	406	1278	1541	1732	1850	1902	1886	1998	2032	1990	1943	1826	1828 ³	1651	1537	1381	1170	349	14625	13675

that the outer layers have a greater angular velocity than the lower seems to be indicated by the observations of the faculæ, which give a somewhat shorter rotation-period than the spot-observations. The accuracy of these observations of the faculæ is, however, difficult to determine, averages only being given. Another fact, pointing in the same direction, is, as we have seen on page 44, that a greater angular velocity is obtained from larger than from smaller spots. Since it now seems very probable that, upon an average, the larger spots occur at higher levels than the smaller, it should also follow from this that the outer layers rotate faster than the inner. This different velocity of rotation is still more evident from spectroscopic observations, especially those made at the Solar Observatory, Mount Wilson¹. As is easily seen, this may be the cause of a small spot being more easily observed on the eastern half of the sun than on the western.

C. The Place of Origin of the Sun-spots. Discussion of the Opinion of Stephani.

Another problem nearly related to that of the distribution is the question of the birthplace of the spots. Since this problem on account of the writings of STEPHANI has recently begun to attract some attention I shall here give the result of an examination, regarding this subject, of the Ledgers published in the Greenwich Observations.

Dr. CARL, *München*, remarks that during the period 1859—1864 spots only exceptionally arose on the side of the sun turned towards the earth. An examination made by STEPHANI of solar photographs taken at the solar observatory in *Cassel* from 1905—1910 gives the result that of the 246 larger spots observed only 20 arose on this side of the sun. As far as I can find STEPHANI does not state, where he places the limit between »large» and »small» spots. The ratio 20 : 246 seems, however, rather strange. A result such as that of STEPHANI would obviously, if proved correct, be of the greatest importance to our ideas of the sun-spots and their origin.

In order to find out the real facts of this case I have examined the first and last observations of all spots observed from 1886—1909 at different distances from the central meridian. I have distinguished between recurrent spots and such spots as have only been observed in one rotation. The result is given in the tables XXXIV—XXXVII. Further, a summary of these observations is given in table

¹ WALTER S. ADAMS: Spectrographic Observations of the Rotation of the Sun. *Astrophysical Journal*, Vol. XXVI, 1907.

—, Preliminary Note on the Rotation of the Sun as determined from the Displacements of the Hydrogen Lines. *Astrophysical Journal*, Vol. XXVII, 1908.

GEORGE E. HALE: Preliminary Note on the Rotation of the Sun as determined from the Motions of the Hydrogen Flocculi. *Astrophysical Journal*, Vol. XXVII, 1908.

TABLE XXXIV.
The first Observation of Non-recurrent Sun-spots.
First Period (1886—1897).

Long. fr. Centr. Merid.	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	Totals
— 80.1° to — 90.0°	3	5	2	1	5	8	15	15	22	13	6	7	102
— 70.1 » — 80.0	18	17	8	2	2	17	43	62	52	47	31	21	320
— 60.1 » — 70.0	11	6	4		7	14	32	47	42	27	20	12	222
— 50.1 » — 60.0	6	2	3	5	4	9	21	23	28	22	20	14	157
— 40.1 » — 50.0	10	6	7	2	5	6	29	37	21	14	11	6	154
— 30.1 » — 40.0	5	2	1	1	1	18	22	21	23	17	10	7	128
— 20.1 » — 30.0	4	3	1	2	4	4	14	26	22	27	11	3	121
— 10.1 » — 20.0	5	2	5	2	4	14	15	27	35	25	11	15	160
— 0.1 » — 10.0	5	4	2	—	—	7	28	28	26	34	14	13	161
0.0 to + 9.9	8	5	1	2	4	8	24	17	29	28	12	11	149
+ 10.0 » + 19.9	3	4	1	—	5	9	27	21	19	18	11	11	129
+ 20.0 » + 29.9	5	2	—	—	2	5	11	21	24	13	9	8	100
+ 30.0 » + 39.9	7	3	5	2	3	10	14	18	20	15	6	9	112
+ 40.0 » + 49.9	4	2	2	1	1	10	11	11	5	16	6	—	69
+ 50.0 » + 59.9	2	1	—	—	2	3	12	9	7	9	4	5	54
+ 60.0 » + 69.9	—	1	—	—	1	3	5	8	7	4	1	3	33
+ 70.0 » + 79.9	—	—	1	1	1	1	2	2	3	3	—	1	15
+ 80.0 » + 89.9	—	—	—	—	—	—	—	2	1	—	—	—	3
Totals	96	65	43	21	51	146	325	395	386	332	183	146	2189

Second Period (1898—1909).

Long. fr. Centr. Merid.	1898	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	Totals
— 80.1° to — 90.0°	5	2	3	1	1	3	4	9	4	11	9	6	58
— 70.1 » — 80.0	16	13	4	4	4	17	36	37	30	17	33	26	235
— 60.1 » — 70.0	13	13	4	1	3	10	20	26	20	22	27	12	171
— 50.1 » — 60.0	9	6	6	—	2	6	15	12	20	12	8	9	105
— 40.1 » — 50.0	10	4	1	1	1	8	14	12	27	13	16	10	117
— 30.1 » — 40.0	5	8	2	—	2	6	18	17	18	18	16	10	120
— 20.1 » — 30.0	9	3	1	1	4	13	16	19	21	11	11	9	118
— 10.1 » — 20.0	13	3	4	2	1	7	16	11	18	18	15	12	120
— 0.1 » — 10.0	6	1	3	—	1	7	19	18	16	19	12	15	117
0.0 to + 9.9	6	5	2	1	1	6	8	16	22	12	14	12	105
+ 10.0 » + 19.9	9	3	—	2	1	5	9	22	11	15	7	8	92
+ 20.0 » + 29.9	5	1	1	—	4	4	11	20	16	11	17	8	98
+ 30.0 » + 39.9	4	1	1	1	4	5	7	10	7	11	10	6	67
+ 40.0 » + 49.9	3	2	1	—	2	9	14	13	7	11	6	6	74
+ 50.0 » + 59.9	7	—	2	—	—	4	2	4	14	5	7	6	51
+ 60.0 » + 69.9	2	1	1	—	—	1	3	7	6	1	5	2	29
+ 70.0 » + 79.9	1	—	—	—	1	—	—	2	1	5	2	1	13
+ 80.0 » + 89.9	—	—	—	—	—	—	—	1	1	—	—	—	2
Totals	123	66	35	13	32	111	212	256	259	212	215	158	1692

TABLE XXXV.

The last Observation of Non-recurrent Sun-spots.

First Period (1886—1897).

Long. fr. Centr. Merid.	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	Totals
— 80.1 to — 90.0	—	—	—	—	—	—	—	—	—	—	—	—	—
— 70.1 » — 80.0	—	—	—	—	—	1	—	4	4	2	1	—	12
— 60.1 » — 70.0	3	1	—	—	2	1	3	8	11	2	4	5	40
— 50.1 » — 60.0	4	2	5	—	3	8	13	10	11	8	11	10	85
— 40.1 » — 50.0	5	5	3	3	3	4	19	20	11	14	6	6	99
— 30.1 » — 40.0	1	3	1	—	2	6	21	20	14	22	5	1	96
— 20.1 » — 30.0	6	4	2	2	1	6	16	11	23	24	8	6	109
— 10.1 » — 20.0	6	7	5	2	2	8	13	31	29	18	10	10	141
— 0.1 » — 10.0	8	4	1	—	1	11	22	36	24	31	13	10	161
0.0 to + 9.9	5	4	3	2	4	6	19	31	31	26	17	9	157
+ 10.0 » + 19.9	6	2	1	—	5	9	37	28	34	29	16	17	184
+ 20.0 » + 29.9	7	3	—	2	2	7	20	27	31	19	13	10	141
+ 30.0 » + 39.9	5	4	6	1	4	12	25	22	31	28	14	8	160
+ 40.0 » + 49.9	4	4	—	1	3	15	17	29	28	20	11	7	139
+ 50.0 » + 59.9	10	9	3	—	8	11	23	31	19	21	12	12	159
+ 60.0 » + 69.9	12	8	4	4	5	15	27	30	32	20	18	13	188
+ 70.0 » + 79.9	12	3	6	4	6	19	39	51	44	34	17	16	251
+ 80.0 » + 89.9	2	2	3	—	—	7	11	6	9	14	7	6	67
Totals	96	65	43	21	51	146	325	395	386	332	183	146	2189

Second Period (1898—1909).

Long. fr. Centr. Merid.	1898	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	Totals
— 80.1 to — 90.0	—	—	—	—	—	—	—	—	—	—	—	—	—
— 70.1 » — 80.0	—	2	—	—	—	—	—	1	2	—	—	1	6
— 60.1 » — 70.0	5	—	—	—	—	4	6	5	5	2	—	3	30
— 50.1 » — 60.0	1	4	1	2	2	2	9	4	4	4	4	1	38
— 40.1 » — 50.0	6	3	—	—	3	5	10	11	18	11	4	7	78
— 30.1 » — 40.0	6	7	1	—	1	5	11	12	15	5	15	6	84
— 20.1 » — 30.0	9	3	1	1	3	9	12	11	12	11	17	8	97
— 10.1 » — 20.0	7	4	—	2	—	6	11	14	13	17	11	9	94
— 0.1 » — 10.0	9	4	4	—	1	11	16	13	15	17	16	8	114
0.0 to + 9.9	6	8	—	1	—	7	14	19	18	14	15	8	110
+ 10.0 » + 19.9	13	5	5	1	3	4	18	25	19	15	16	13	137
+ 20.0 » + 29.9	10	3	4	1	3	8	12	19	20	15	18	12	125
+ 30.0 » + 39.9	5	1	4	1	3	6	17	10	23	13	23	14	120
+ 40.0 » + 49.9	9	6	2	—	4	7	12	28	11	15	14	15	123
+ 50.0 » + 59.9	10	4	3	1	2	5	13	24	23	15	16	11	127
+ 60.0 » + 69.9	12	7	5	1	1	14	23	30	27	22	10	16	168
+ 70.0 » + 79.9	15	5	2	2	3	16	21	23	22	30	31	21	191
+ 80.0 » + 89.9	—	—	3	—	3	2	7	7	12	6	5	5	50
Totals	123	66	35	13	32	111	212	256	259	212	215	158	1692

TABLE XXXVI.

The first Observation of Recurrent Sun-spots.

First Period (1886—1897).

Long. fr. Centr. Merid.	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	Totals
— 80.1° to — 90.0°	2	1	—	—	—	3	1	12	6	3	1	2	31
— 70.1 » — 80.0	—	1	2	1	2	6	9	10	14	13	4	5	67
— 60.1 » — 70.0	—	1	—	—	—	4	5	6	2	4	1	1	25
— 50.1 » — 60.0	—	—	—	—	—	2	5	3	1	5	—	—	16
— 40.1 » — 50.0	1	—	—	1	—	3	1	2	1	—	2	—	11
— 30.1 » — 40.0	1	1	—	—	—	—	2	7	—	1	—	—	12
— 20.1 » — 30.0	—	—	—	—	—	3	4	5	1	2	—	—	15
— 10.1 » — 20.0	1	—	—	—	—	1	1	4	3	3	1	—	14
— 0.1 » — 10.0	2	—	—	—	—	1	—	2	4	—	2	1	12
0.0 to + 9.9	—	—	1	—	—	—	2	2	4	—	3	1	13
+ 10.0 » + 19.9	—	—	—	—	—	3	1	1	1	—	1	2	9
+ 20.0 » + 29.9	—	—	1	1	—	1	—	3	1	2	—	—	9
+ 30.0 » + 39.9	—	—	—	—	—	2	5	1	3	—	—	—	11
+ 40.0 » + 49.9	—	—	—	—	—	1	—	4	2	1	2	—	10
+ 50.0 » + 59.9	1	—	—	1	—	1	2	1	1	2	1	—	10
+ 60.0 » + 69.9	1	1	—	—	—	1	3	1	1	—	2	—	10
+ 70.0 » + 79.9	—	—	—	—	—	—	—	1	1	—	—	—	2
+ 80.0 » + 89.9	—	—	—	—	—	—	—	—	1	—	1	—	2
Totals	10	5	4	4	2	32	41	65	47	36	21	12	279

Second Period (1898—1909).

Long. fr. Centr. Merid.	1898	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	Totals
— 80.1° to — 90.0°	2	—	—	—	—	4	2	4	2	4	1	5	24
— 70.1 » — 80.0	2	2	1	1	1	5	6	9	9	10	4	5	55
— 60.1 » — 70.0	—	—	—	—	—	1	2	1	1	2	2	1	10
— 50.1 » — 60.0	2	—	—	—	—	2	—	1	5	3	4	2	19
— 40.1 » — 50.0	—	—	—	—	—	—	1	3	2	4	2	1	13
— 30.1 » — 40.0	—	—	—	—	1	—	1	2	6	1	1	4	16
— 20.1 » — 30.0	1	—	1	—	—	—	3	2	1	4	1	—	13
— 10.1 » — 20.0	1	—	1	—	—	—	2	1	1	1	1	—	8
— 0.1 » — 10.0	1	1	—	—	—	2	1	2	1	—	2	2	12
0.0 to + 9.9	1	1	—	—	—	1	1	2	3	3	2	1	15
+ 10.0 » + 19.9	—	—	1	—	—	—	3	1	2	5	2	4	18
+ 20.0 » + 29.9	1	—	—	—	—	—	1	2	1	3	—	—	8
+ 30.0 » + 39.9	—	—	—	—	—	1	1	1	2	1	3	2	11
+ 40.0 » + 49.9	—	—	—	—	—	—	—	1	—	1	1	1	4
+ 50.0 » + 59.9	—	—	1	—	—	—	2	4	1	3	3	—	13
+ 60.0 » + 69.9	—	—	1	—	—	—	2	1	—	1	1	—	6
+ 70.0 » + 79.9	—	—	—	—	—	—	1	2	—	1	—	1	5
+ 80.0 » + 89.9	—	—	—	—	—	—	—	—	—	—	1	—	1
Totals	11	4	6	1	2	16	29	39	36	47	31	29	251

TABLE XXXVII.
The last Observation of Recurrent Sun-spots.
First Period (1886—1897).

Long. fr. Centr. Merid.	1886	1887	1888	1889	1890	1891	1892	1893	1894	1895	1896	1897	Totals
— 80.1° to — 90.0°	—	—	—	—	—	—	—	—	—	—	—	—	—
— 70.1 » — 80.0	—	—	1	—	—	—	—	1	1	—	—	—	3
— 60.1 » — 70.0	—	1	1	—	—	—	2	1	4	—	—	—	9
— 50.1 » — 60.0	—	—	—	—	1	2	—	2	2	2	1	—	10
— 40.1 » — 50.0	—	—	—	—	—	2	—	4	1	—	—	1	8
— 30.1 » — 40.0	1	—	1	—	—	1	1	3	—	2	2	—	11
— 20.1 » — 30.0	—	—	—	—	—	1	1	3	2	4	2	1	14
— 10.1 » — 20.0	—	1	1	—	—	—	1	5	4	1	1	2	16
— 0.1 » — 10.0	—	1	—	—	—	3	1	1	3	1	2	—	12
0.0 to + 9.9	—	—	—	—	—	2	3	5	1	1	—	—	12
+ 10.0 » + 19.9	2	—	—	—	—	2	3	4	3	2	2	—	18
+ 20.0 » + 29.9	—	—	—	—	—	1	3	—	6	2	—	—	12
+ 30.0 » + 39.9	—	—	—	—	—	3	3	1	3	3	3	—	16
+ 40.0 » + 49.9	1	—	—	—	1	1	1	5	2	4	1	—	16
+ 50.0 » + 59.9	—	—	—	1	—	1	3	4	3	1	—	1	14
+ 60.0 » + 69.9	1	1	—	2	—	4	4	7	2	2	—	—	23
+ 70.0 » + 79.9	2	1	—	1	—	9	12	12	6	9	7	7	66
+ 80.0 » + 89.9	3	—	—	—	—	—	3	7	4	2	—	—	19
Totals	10	5	4	4	2	32	41	65	47	36	21	12	279

Second Period (1898—1909).

Long. fr. Centr. Merid.	1898	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	Totals
— 80.1° to — 90.0°	—	—	—	—	—	—	—	—	1	—	—	—	1
— 70.1 » — 80.0	—	—	—	—	—	1	—	—	1	—	—	—	2
— 60.1 » — 70.0	—	—	—	—	—	2	1	—	—	2	—	1	6
— 50.1 » — 60.0	—	1	1	—	2	—	2	1	2	1	—	4	14
— 40.1 » — 50.0	—	—	—	—	—	—	—	2	3	3	1	—	9
— 30.1 » — 40.0	1	—	—	—	—	—	1	3	2	3	1	—	11
— 20.1 » — 30.0	—	1	1	—	—	—	2	3	—	1	2	2	12
— 10.1 » — 20.0	—	—	—	—	—	1	3	3	2	2	—	2	13
— 0.1 » — 10.0	—	—	—	—	—	1	—	1	2	6	3	1	14
0.0 to + 9.9	1	—	—	—	—	1	1	3	—	1	1	1	9
+ 10.0 » + 19.9	—	—	—	—	—	3	1	—	2	1	—	3	10
+ 20.0 » + 29.9	—	—	1	—	—	1	4	3	4	2	2	—	17
+ 30.0 » + 39.9	—	1	—	—	—	2	2	4	1	4	2	—	16
+ 40.0 » + 49.9	2	—	—	—	—	—	2	—	2	5	3	2	16
+ 50.0 » + 59.9	—	—	—	—	—	—	1	1	3	3	2	2	12
+ 60.0 » + 69.9	—	—	2	—	—	1	3	7	—	7	8	5	33
+ 70.0 » + 79.9	6	—	—	1	—	2	6	6	6	5	6	2	40
+ 80.0 » + 89.9	1	1	1	—	—	1	—	2	5	1	—	4	16
Totals	11	4	6	1	2	16	29	39	36	47	31	29	251

TABLE XXXVIII.
First and last Observations of the Sun-spots.
First Period (1886—1897).

Longitude from Central Meridian	Recurrent Spots		Non-recurrent Spots		$n_1 - n_2$	$n_3 - n_4$
	First Observ. n_1	Last Observ. n_2	First Observ. n_3	Last Observ. n_4		
— 80.1 to — 90.0	31	0	102	0	+ 31	+ 102
— 70.1 » — 80.0	67	3	320	12	+ 64	+ 308
— 60.1 » — 70.0	25	9	222	40	+ 16	+ 182
— 50.1 » — 60.0	16	10	157	85	+ 6	+ 72
— 40.1 » — 50.0	11	8	154	99	+ 3	+ 55
— 30.1 » — 40.0	12	11	128	96	+ 1	+ 32
— 20.1 » — 30.0	15	14	121	109	+ 1	+ 12
— 10.1 » — 20.0	14	16	160	141	— 2	+ 19
— 0.1 » — 10.0	12	12	161	161	0	0
0.0 to + 9.9	13	12	149	157	+ 1	— 8
+ 10.0 » + 19.9	9	18	129	184	— 9	— 55
+ 20.0 » + 29.9	9	12	100	141	— 3	— 41
+ 30.0 » + 39.9	11	16	112	160	— 5	— 48
+ 40.0 » + 49.9	10	16	69	139	— 6	— 70
+ 50.0 » + 59.9	10	14	54	159	— 4	— 105
+ 60.0 » + 69.9	10	23	33	188	— 13	— 155
+ 70.0 » + 79.9	2	66	15	251	— 64	— 236
+ 80.0 » + 89.9	2	19	3	67	— 17	— 64
Totals	279	279	2189	2189	—	—

Second Period (1898—1909).

Longitude from Central Meridian	Recurrent Spots		Non-recurrent Spots		$n_1 - n_2$	$n_3 - n_4$
	First Observ. n_1	Last Observ. n_2	First Observ. n_3	Last Observ. n_4		
— 80.1 to — 90.0	24	1	58	0	+ 23	+ 58
— 70.1 » — 80.0	55	2	235	6	+ 53	+ 229
— 60.1 » — 70.0	10	6	171	30	+ 4	+ 141
— 50.1 » — 60.0	19	14	105	38	+ 5	+ 67
— 40.1 » — 50.0	13	9	117	78	+ 4	+ 39
— 30.1 » — 40.0	16	11	120	84	+ 5	+ 36
— 20.1 » — 30.0	13	12	118	97	+ 1	+ 21
— 10.1 » — 20.0	8	13	120	94	— 5	+ 26
— 0.1 » — 10.0	12	14	117	114	— 2	+ 3
0.0 to + 9.9	15	9	105	110	+ 6	— 5
+ 10.0 » + 19.9	18	10	92	137	+ 8	— 45
+ 20.0 » + 29.9	8	17	98	125	— 9	— 27
+ 30.0 » + 39.9	11	16	67	120	— 5	— 53
+ 40.0 » + 49.9	4	16	74	123	— 12	— 49
+ 50.0 » + 59.9	13	12	51	127	+ 1	— 76
+ 60.0 » + 69.9	6	33	29	168	— 27	— 139
+ 70.0 » + 79.9	5	40	13	191	— 35	— 178
+ 80.0 » + 89.9	1	16	2	50	— 15	— 48
Totals	251	251	1692	1692	—	—

XXXVIII. In the last two columns of this table, I have given the difference between the number of the first and last observations at different distances from the central meridian. This difference is almost thoroughly positive on the eastern and negative on the western half of the sun. In reality, the same number of spots undoubtedly arises and disappears within each of these zones. This also proves to be approximately the case with recurrent spots between about -60° and $+60^\circ$. These spots are, upon an average, certainly larger than spots observed only in one rotation. From the table we see that the number of arising and disappearing spots of the latter kind is the same only in the zone between -10° and $+10^\circ$. During the first

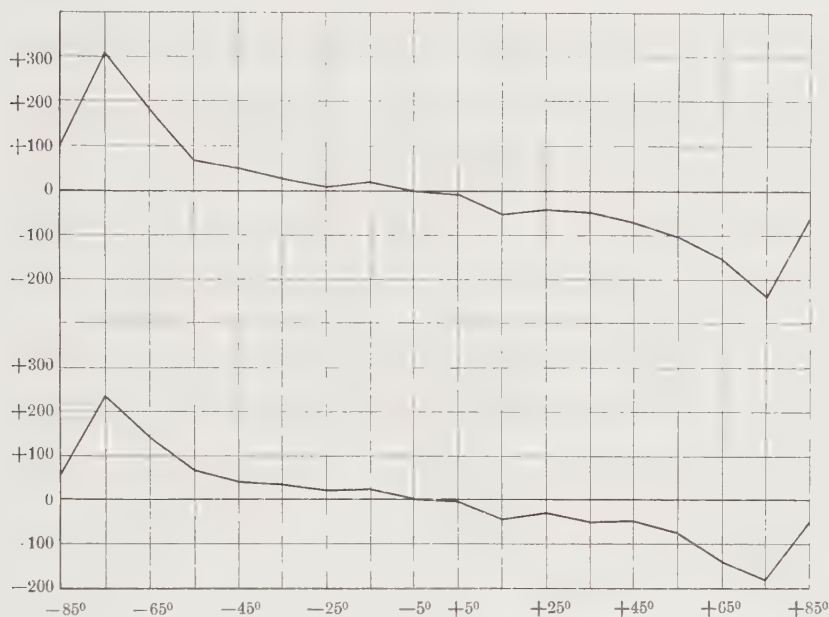


Fig. 8. The Difference between the Number of first and last Observations of Sun-spots at different Distances from the Central Meridian.

period (1886—1897), 310 such spots arose within this zone and 318 disappeared. During the second the corresponding numbers were 222 and 224.

Is the opinion of STEPHANI confirmed by this table? Without entering into any further discussion of the table in question, I will merely call attention to the fact that the number of first observations of recurrent spots between -60° and $+60^\circ$ during the first period amounts to 0.51 and during the second to 0.60 of all recurrent spots that have been observed. The corresponding numbers for non-recurrent spots are 0.68 and 0.70. That these numbers are greater than $\frac{1}{2}$ may be due to the non-observance of a great number of the spots arising on the other side of the sun. That these ratios are greater for non-recurrent than for recurrent

spots, may be easily explained by the fact that the latter, upon an average, exist for a longer time than the former. Hence it is obvious that the opinion expressed by STEPHANI is not confirmed by these observations. In fig. 8 I have given a graphical representation of the difference between the number of arising and disappearing non-recurrent spots at different distances from the central meridian. As is seen from the figure, there exists a close agreement between the observations from the two periods.



CHAPTER VII.

The Effect of a Refraction in the Solar Atmosphere.

A. The Longitude Motions of the Spots at different Distances from the Central Meridian.

The existence of a refraction in the solar atmosphere causing the velocity of the spots apparently to decrease with increasing distance from the centre, has been previously pointed out. A correction for this effect of refraction is applied to the co-ordinates of sun-spots published in *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*. Any such correction, which must in fact be rather approximately determined, has, on the contrary, not been applied to the measurements of the photographs taken at *Greenwich*, which, as before stated, are the basis of the present investigation.

Since the size of the refraction is dependent on the distance of the spot from the centre of the sun, it may evidently be determined by an examination of the motion of the spots at different distances from this centre. As I did not intend at first to investigate this problem, I have not completely accomplished such a division of the material of observation. Some remarks on this question may, however, be of some interest.

We have found in the preceding chapter that the two halves of the sun in some respects presented certain dissimilarities. This induced me to examine whether the observations of the spots on both sides of the central meridian should give different values of the angular velocity. The differences between the mean values of the longitude motions on the western and eastern halves amount, for the two hemispheres, during the first period to $+0.201$ and $+0.195$, during the second to $+0.244$ and $+0.240$. The mean errors only amounting to 0.050 , this difference cannot be accidental.

On account of this difference I made a further division of the material according to the distance from the central meridian. In this division I have taken both periods as well as both hemispheres together. The results of the computations are brought together in table XXXIX. As seen from this table, there exists an evident

rate in the averages of each particular zone. This appears still more obvious if we compute the means of all the zones. From these means we may infer that the angular velocity determined from the motions of the spots is dependent on the distance from the centre of the sun. Moreover, we see that there exists a distinct difference between the two halves. Using the motion in longitude as ordinate and the distance from the central meridian as abscissa, the curve obtained will not be symmetrical about the central meridian, its maximum being situated on the western side of this meridian.

TABLE XXXIX.

Mean Values of the Motions in Longitude at different Distances from the Central Meridian 1886–1909.

Latitude-zones	$-60^{\circ}.0 < l < -50^{\circ}.1$		$-50^{\circ}.0 < l < -40^{\circ}.1$		$-40^{\circ}.0 < l < -30^{\circ}.1$		$-30^{\circ}.0 < l < -20^{\circ}.1$		$-20^{\circ}.0 < l < -10^{\circ}.1$		$-10^{\circ}.0 < l < -0^{\circ}.1$	
	<i>n</i>	x_0	<i>n</i>	x_0	<i>n</i>	x_0	<i>n</i>	x_0	<i>n</i>	x_0	<i>n</i>	x_0
N_1 and S_1	121	+0.880	105	+1.081	110	+0.973	120	+1.025	132	+0.955	116	+1.319
N_2 » S_2	349	+0.769	385	+0.861	391	+0.917	370	+0.806	388	+1.023	382	+1.327
N_3 » S_3	478	+0.410	511	+0.608	502	+0.709	509	+0.787	487	+0.531	504	+0.803
N_4 » S_4	335	+0.022	371	+0.255	399	+0.510	375	+0.455	399	+0.605	393	+0.605
N_5 » S_5	187	+0.131	191	+0.149	202	-0.064	175	-0.049	218	+0.294	213	+0.486
Means	1470	+0.410	1563	+0.562	1604	+0.631	1549	+0.635	1624	+0.669	1608	+0.874

Latitude-zones	$0^{\circ}.0 < l < +9^{\circ}.9$		$+10^{\circ}.0 < l < +19^{\circ}.9$		$+20^{\circ}.0 < l < +29^{\circ}.9$		$+30^{\circ}.0 < l < +39^{\circ}.9$		$+40^{\circ}.0 < l < +49^{\circ}.9$	
	<i>n</i>	x_0	<i>n</i>	x_0	<i>n</i>	x_0	<i>n</i>	x_0	<i>n</i>	x_0
N_1 and S_1	130	+1.377	112	+1.366	106	+1.113	116	+1.190	66	+0.773
N_2 » S_2	372	+0.943	355	+1.035	335	+1.145	329	+1.129	178	+0.826
N_3 » S_3	508	+1.006	451	+0.730	465	+0.943	432	+0.931	224	+0.228
N_4 » S_4	393	+0.795	352	+0.457	342	+0.801	355	+0.711	192	+0.104
N_5 » S_5	194	+0.428	185	+0.797	170	+0.406	162	+0.586	102	-0.245
Means	1597	+0.899	1455	+0.796	1418	+0.905	1394	+0.903	762	+0.320

This strange fact might possibly be explained in the following way, without supposing a different refraction. As we have previously seen, the small spots are more difficult to observe on the western half of the sun than on the eastern. These small spots, part of which are excluded in determining the motions in longitude on the western half, give, however, a smaller angular velocity than the large ones. From this it follows that we ought to have a greater angular velocity from observations on the western half of the sun than from those on the eastern, a fact in close agreement with the results obtained.

B. On the Possibility of two Drifts in the Motion of the Sun-spots.

As mentioned in the introduction, HIRAYAMA, in an examination of the table of correlation between latitude and synodic rotation period given by MAUNDER, has supposed two apparent drifts of sun-spots as a tentative explanation of the skewness in the distribution of the rotation periods. The method employed by HIRAYAMA in dissecting the frequency-curves into two components is exclusively graphical, and thus we cannot attach to it any great accuracy. From the graphical representation given by HIRAYAMA in his paper it seems as if all the curves of distribution were attended by a very considerable positive skewness.

In order to examine the distribution analytically I have computed the characteristics of these curves. The result of the computations is given in table XL, where T_0 designates the mean of the synodic rotation periods. From this it follows that the skewness of the frequency-curves of HIRAYAMA, drawn on an arbitrary scale, is not so large as it appears on a cursory observation. And since the mean errors which are also given in the table are approximately of the same size as the skewness, it seems somewhat ungrounded to make a division into components in such a way as is done by HIRAYAMA. Furthermore, HIRAYAMA does not take the excess, throughout positive, into consideration, an omission which to a certain degree makes this division illusory.

TABLE XL.
Characteristics of the Distribution of synodic Rotation Periods.

Latitude	n	T_0	σ	S	E
$+25^0 < \beta < +30^0$	36	27.578 ± 0.179	1.074 ± 0.127	$+0.056 \pm 0.332$	$+0.009 \pm 0.102$
$+20 < \beta < +25$	115	27.312 ± 0.094	1.014 ± 0.067	-0.167 ± 0.180	$+0.172 \pm 0.057$
$+15 < \beta < +20$	188	26.819 ± 0.057	0.782 ± 0.040	$+0.392 \pm 0.141$	$+0.067 \pm 0.045$
$+10 < \beta < +15$	290	26.755 ± 0.048	0.818 ± 0.034	-0.052 ± 0.113	$+0.201 \pm 0.036$
$+5 < \beta < +10$	154	26.466 ± 0.060	0.744 ± 0.065	$+0.185 \pm 0.156$	$+0.044 \pm 0.049$
$-5 < \beta < +5$	133	26.354 ± 0.069	0.800 ± 0.049	$+0.166 \pm 0.168$	$+0.022 \pm 0.053$
$-5 > \beta > -10$	240	26.673 ± 0.050	0.776 ± 0.035	$+0.319 \pm 0.125$	$+0.088 \pm 0.040$
$-10 > \beta > -15$	294	26.697 ± 0.051	0.871 ± 0.036	$+0.191 \pm 0.113$	$+0.177 \pm 0.036$
$-15 > \beta > -20$	252	26.944 ± 0.060	0.945 ± 0.042	$+0.126 \pm 0.122$	$+0.138 \pm 0.039$
$-20 > \beta > -25$	110	27.136 ± 0.080	0.839 ± 0.056	$+0.109 \pm 0.184$	-0.006 ± 0.058
$-20 > \beta > -30$	44	27.509 ± 0.147	0.974 ± 0.104	$+0.259 \pm 0.291$	$+0.118 \pm 0.092$

As seen from the previous investigation, the skewness of the distribution of the motions of the spots is proved to be inconsiderable. It is now obvious that if the distribution of the values of a variable x is normal, the distribution of $1:x$ will be attended by a *negative* skewness. We ought thus, upon an average, to expect a negative skewness in the distribution of the rotation periods of the spots instead of a positive, as is the case here. Here, however, we must take into consideration the fact that MAUNDER has determined the synodic rotation-period from spots exi-

sting at least for six consecutive days. As seen from table XXXIX the spots on the western half of the sun give a shorter rotation-period than those on the eastern, and in this fact we possibly have the explanation of the positive skewness. Such a skewness might arise if a greater number of spots were observed on the eastern than on the western half, which we know to be the case.

Since, however, the mean errors of the skewnesses of HIRAYAMA's curves are of about the same size as the skewnesses themselves, we can, however, scarcely attach any great importance to the assumption of two drifts in the motion of the sun-spots.

Summary.

The present investigation of the motions of the sun-spots is based on measurements of the co-ordinates of the spots during the period 1886—1909 published in the Greenwich Observations. From these measurements the motions of the spots have been computed in minutes of arc per hour. All such spots are included in the investigation as have been observed within 60° from the central meridian for at least two consecutive days, and the motion of which in longitude and latitude is smaller than $10'$ and $5'$ respectively, corresponding to a linear velocity of 82 and 41 km. per hour.

I shall here give a summary of the results obtained, based, for the most part, on no less than 16742 observations of longitude and latitude motions.

1. The daily mean area of the sun-spots Δ expressed in millionths of the sun's visible disk may be computed with considerable accuracy from the relative numbers r of WOLF by means of the formula

$$\Delta = 7.466 \cdot r^{1.185} \quad (\text{Page 5}).$$

2. The »law of SPÖRER» is positively confirmed from the table on page 14.
3. The distribution of the motions in longitude as well as in latitude is attended by a considerable positive excess, which still remains unexplained. The skewness, on the contrary, is, upon an average, equal to zero. There exists possibly an inconsiderable decrease of the skewness of the motions in longitude with increasing distance from the equator (Table XII). The dispersion of the longitude motions is double the size of the dispersion of the motions in latitude. (Page 28, 33, 59).
4. On account of the considerable dispersion, the angular velocity cannot be determined with great accuracy. The daily rate of the sun's rotation is obtained from the following formulæ:

First Period (1886—1900): $V = 14^{\circ}.577 + 0^{\circ}.096 \sin \beta - 2^{\circ}.322 \sin^2 \beta$;

Second Period (1898—1909): $V = 14^{\circ}.489 - 0^{\circ}.076 \sin \beta - 2^{\circ}.156 \sin^2 \beta$.

By way of comparison the following formulæ may be mentioned here.

CARRINGTON (1853—1861): $V = 14^{\circ}.417 - 2^{\circ}.750 \sin^2 \beta$;

SPÖRER (1861—1871): $V = 8^{\circ}.532 + 5^{\circ}.798 \cos \beta$;

MAUNDER (1879—1901): $V = 14^{\circ}.443 - 2^{\circ}.133 \sin^2 \beta$.

The angular velocity of the northern hemisphere is positively greater during the first period (1886—1897) than during the second (1898—1909). On the contrary, no such difference seems to exist for the southern hemisphere. (Page 38, 39).

5. The large spots give a greater angular velocity than the small ones, a fact which may be explained in supposing the smaller spots generally to occur in deeper lying layers than the larger. (Page 44).
6. Spots on the western half of the sun give a greater angular velocity than spots on the eastern. (Page 90).
7. The dispersion of the motions of the spots diminishes with increasing spot-area. The relation between σ and ω is approximately expressed by

$$\omega\sigma^2 = \text{constant.} \quad (\text{Page 64}).$$

8. The quotient between the dispersion of the motions in latitude and longitude seems to increase with increasing spot area. (Table XXIX on page 62).
9. The latitude drifts towards the poles and the equator assumed by CARRINGTON are not in the least indicated. On the contrary, there exists an inconsiderable northern drift in both hemispheres. (Page 46).
10. In the latitude motions there exists an evident annual period, which is most conspicuous during the years 1886—1897. The computed corrections of the sun's axis show that the inclination between the sun's equator and the ecliptic has shown an increase of about 9' from the first period to the second. (Page 56).
11. Between the motions in longitude and latitude there exists a negative correlation in the northern and a positive in the southern hemisphere. The angle between the line of symmetry of the frequency-surfaces and the equator seems to increase with increasing latitude. (Page 72).
12. There exists a pronounced preponderance of spots on the eastern half of the sun. (Page 79).
13. The opinion expressed by STEPHANI that about 90 % of the »large» spots arise on the side of the sun turned from the earth, is not in the least confirmed by the observations used in this investigation. (Page 8).
14. There seems to be no reason for HIRAYAMA's assumption of two different drifts of sun-spots. (Page 91).

* * *

It is with special pleasure that I take this opportunity of expressing my gratitude to Professor C. V. L. CHARLIER, not only for the suggestion of this and other works but also for the warm interest with which he has followed the progress of the present investigation.

I also wish to convey my thanks to the *Royal Physiographical Society of Lund* for the grant which has been made to me, for the furtherance of this work, out of the Society's RETZIUS Fund.

Table of Contents.

	Page
Introduction. Historical Summary	3
The Periodicity of the Sun-spots	3
The Motions of the Sun spots	9
The Distribution of the Sun-spots	13
Chapter I. The Material of Observation and its first Treatment	16
Chapter II. The Longitude Motions of the Sun-spots	21
A. The Classification of the Observations.....	21
B. Characteristics of the Motions in Longitude	23
Summary of the Method employed	23
Mean and Dispersion	27
Skewness and Excess	28
C. The Rotation Period of the Sun, computed from the Longitude Motions of the Spots	32
D. The Motions in Longitude of Spots of different Sizes.....	42
Chapter III. The Latitude Motions of the Sun spots	46
A. Characteristics of the Motions in Latitude	46
B. On the Position of the Sun's Axis	52
Chapter IV. The Dispersion of the Motions for different Spot-sizes	61
Chapter V. Correlation between the Longitude and Latitude Motions of the Sun-spots.....	65
Chapter VI. The Distribution of Sun-spots on the Sun's Surface	77
A. The Distribution of Spots in the two Hemispheres.....	77
B. The Distribution of Spots on both Halves of the Sun	79
C. The Place of Origin of the Sun-spots. Discussion of the Opinion of Stephani	81
Chapter VII. The Effect of a Refraction in the Solar Atmosphere	89
A. The Longitude Motions of the Spots at different Distances from the Central Meridian	89
B. On the Possibility of two Drifts in the Motions of the Sun-spots	91
Summary	93

Tables.

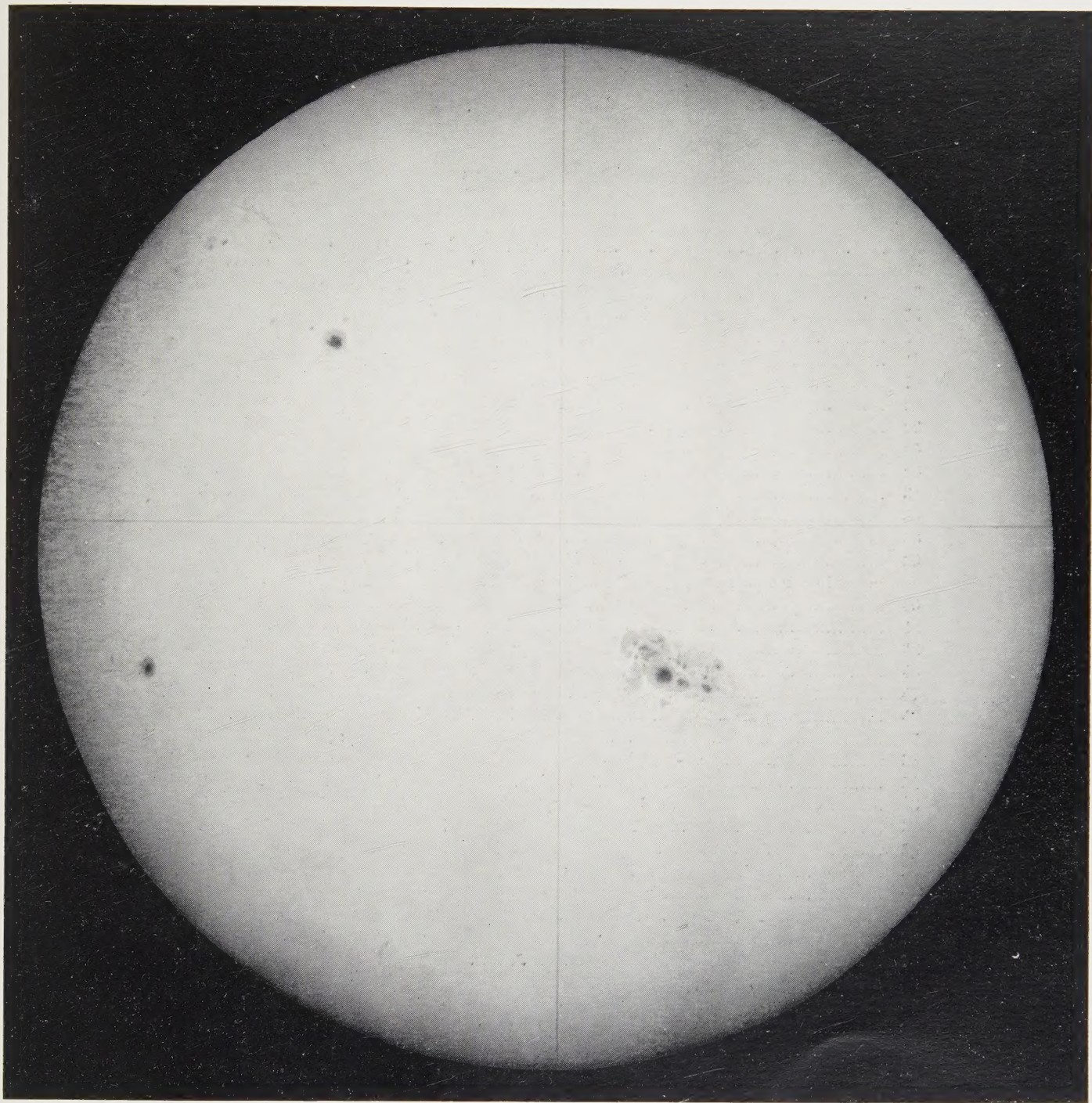
Table	Page
I. Horrebow's Sun-spot Observations	4
II. Schwabe's Observations 1823—43.....	4
III. The Relative Numbers and Spot-areas 1874—1910	5
IV. The Distribution of the Sun-spots in Latitude 1886—1909	14
V. Days with Solar Photographs.....	16
VI. Extract from the Ledgers of Groups of Sun-spots	17
VII. Daily Mean Areas of Sun-spots.....	18
VIII. The Number of Observations in different Years.....	22
IX. The Number of Observations at different Latitudes	22
X. The Motions in Longitude of the Spots at different Latitudes.....	25
XI. Characteristics of the Motions in Longitude.....	28
XII. Correlation between $\Delta\lambda \cos \beta$ and $\log \Delta$	31
XIII. Characteristics of the Motions in Longitude 1886—1909	33
XIV. The angular Velocity at different Latitudes	36
XV. Computed angular Velocities at different Latitudes	39
XVI. Means and Dispersions of the Motions in Longitude and Latitude for different Spot-sizes.....	40
XVII. The Values of α_0 for different Spot-sizes 1886—1909 ..	42
XVIII. The Values of σ_x for different Spot-sizes 1886—1909	42
XIX. The Values of $V - 35'.461$ for different Spot-sizes 1886—1909	43
XX. The Motions in Latitude of the Spots at different Latitudes	47
XXI. Characteristics of the Motions in Latitude	49
XXII. The Values of $\sigma_y : \sigma_x$ at different Latitudes	49
XXIII. Characteristics of the Latitude Motions for different Spot-sizes	50
XXIV. Proper Motion in Latitude of Recurrent Spot Groups	51
XXV. Values of $\Delta L \sin I$ and ΔI according to Dyson and Maunder	53
XXVI. The Motions in Latitude at different Times in the Year	54
XXVII. Mean Values of the Motions in Latitude at different Times in the Year	56
XXVIII. Values of u_0 , $M(\sin u_0)$ and $M(\cos u_0)$ at different Times in the Year	58
XXIX. The Dispersion of the Motions for different Spot-sizes.....	62
XXX. Correlation between the Motions in Longitude and Latitude 1886—1909.....	66
XXXI. Coefficient of Correlation and Line of Symmetry	73
XXXII. Characteristics of the Spot-sizes	78
XXXIII. Number of Spots or Groups of Spots at different Distances from the Central Meridian	80
XXXIV. The first Observation of Non-recurrent Sun-spots ..	82
XXXV. The last Observation of Non-recurrent Sun-spots ..	83
XXXVI. The first Observation of Recurrent Sun-spots	84
XXXVII. The last Observation of Recurrent Sun-spots	85
XXXVIII. First and last Observations of the Sun-spots ..	86
XXXIX. Mean Values of the Motions in Longitude at different Distances from the Central Meridian 1886—1909.....	90
XI. Characteristics of the Distribution of synodic Rotation Periods	91

Figures.

	Page
Fig. 1. Daily Mean Area of Sun-spots 1874—1910	2
2. The Distribution of the Motions in Longitude	32
3. The angular Velocity of the Sun at different Latitudes	38
4. The angular Velocity V at different Latitudes as a Function of ω	44
5. Proper Motion in Latitude of Recurrent Spot Groups	51
6. Proper Motion in Latitude of Recurrent Spot Groups compared with a normal Distribution	52
7. The Dispersion in Longitude and Latitude as Function of the Spot-area	63
8. The Difference between the Number of first and last Observations at different Distances from the Central Meridian.....	87

Plate.

Photograph of the Sun taken at the Royal Observatory, Greenwich, at $12^h 12^m 22^s$ Greenwich Mean Time, February 5, 1905.



Photograph of the Sun taken at the Royal Observatory, Greenwich,
at 12^h12^m22^s Greenwich Mean Time, February 5, 1905 (Page 16).

